Standing on the Shoulders of Dwarfs: Dominant Firms and Innovation Incentives

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Abstract. We develop a dynamic innovation model with three important features: (a) asymmetry between large and small firms (“giants” and “dwarfs”); (b) technology transfer by acquisition; and (c) the distinction between radical innovation (compete for the market) and incremental innovation (compete within the market). We provide conditions such that (a) greater asymmetry between giant and dwarfs decreases incremental innovation but increases radical innovation; and (b) allowing for technology transfer increases incremental innovation but decreases radical innovation.

These results have several policy implications, including: (a) with weak markets for technology, a soft antitrust policy toward dominant firms leads to an increase in radical innovation but a decrease in incremental innovation; (b) a merger policy that restricts the acquisition of fringe firms by dominant firms leads to lower incremental innovation rates and higher radical innovation rates; (c) the effect of IP protection on innovation is mixed: by increasing the prize from patenting, it increases incremental innovation; but, by improving the market for technology, it reduces the rate of radical innovation.
1. Introduction

As Segal and Whinston (2007) aptly point out, “over the last two decades a large share of the economy — the so-called ‘new economy’ — has emerged ... in which innovation is a critical determinant of competitive outcomes and welfare.” A salient feature of many of these industries — especial in the digital space — is the presence of a dominant firm: examples include IBM in the 1980s; Microsoft in the 1990s; Google and Facebook in the 2000s; and Intel since the 1980s. In these industries, the distinction between technology leadership and market leadership becomes relevant. For example, while Intel is clearly the market leader in the microprocessor industry (in terms of production capacity, brand recognition, and so forth), there have been times when AMD has taken the technology lead (in terms of processor speed, for example).

An additional salient feature of many of these industries is the phenomenon of technology transfer — typically by acquisition — which assures the dominant firm remains on the technology edge: a significant number of today's most popular and successful products originated with smaller companies which were later gobbled up by one of the big players. A very partial list includes Google acquiring Applied Semantics (Adsense), Android and YouTube; Microsoft acquiring Hotmail and Forethought (Powerpoint); and Facebook acquiring Instagram.

In this setting, a number of positive and normative questions arise. For example:

- How does the antitrust treatment of dominant firms — in cases such as US vs Microsoft or DG Comp vs Intel — influence the rates of incremental and radical innovation?
- Markets for technology (MFT) vary significantly across industries. How do better MFT influence incremental and radical innovation?
- Is a tougher merger policy good for incremental innovation? Radical innovation?

This paper tackles these and related questions by developing a model of innovation competition with (a) a dominant and a fringe firm; (b) the possibility of technology transfer; and (c) the explicit distinction between incremental and radical innovation.

Model summary. In line with the technology-ladder approach to innovation, we define technology leader (resp. laggard) as the firm whose technology level is higher (resp. lower). The novel aspect of the model is the characterization of market asymmetries (“giants” and “dwarfs”). We define a dominant firm (“giant”) as a one that, for a given technology level, receives greater market payoffs (because, for example, it possesses complementary assets that enhance the value of its technology). For example, Powerpoint has greater market value in the hands of Microsoft than in the hands of a smaller firm such as Forethought (its original developer); or: Instagram creates more value when integrated in Facebook than as an independent startup. We refer to the dominant firm’s rival as the fringe firm (“dwarf”).

Most of the paper assumes that the identity of the dominant firm is fixed and focuses on incremental innovation (or simply innovation), whereby a technology laggard becomes a technology leader. Section 4 extends the analysis to radical innovation, whereby the fringe firm becomes the dominant firm.

We consider an continuous-time innovation game: at each moment in time, firms (a) receive a flow of product market payoffs according to their current industry and technology state; (b) choose (at a cost) their innovation rate. With some probability, innovation is
successful, leading to a change in state. There are two possible states: either the dominant firm is the technology leader or the fringe firm is the technology leader.\textsuperscript{1}

The possibility of technology transfer is considered in a model variation where firms have the ability to negotiate a change in state: by paying a transfer price $p$, the technology laggard becomes a technology leader (and vice-versa). We assume efficient bargaining, which implies technology transfer takes place when the dominant firm is a technology laggard and price is determined by splitting gains from an agreement.

Finally, the possibility of radical innovation is considered in an extension where a fringe firm becomes a dominant firm with a hazard rate that depends on its directed innovation effort.

The paper determines the equilibrium of the dynamic innovation game and studies its comparative statics with respect to: (a) the degree of market dominance: a greater value of the parameter $\alpha$ implies a greater market profit for the dominant firm, all else constant constant; and a lower market profit for the fringe firm; (b) the efficiency of markets for technology: two extreme possibilities are considered, no technology transfer and efficient technology transfer.

\textbf{Main results summary.} The analysis leads to three main results. First, absent technology transfer, an increase in the degree of firm dominance leads to a decrease in the average incremental rate. (Unless otherwise noted, by “innovation” we mean incremental innovation, the main focus in most of the paper.) This is perhaps the most complex result as it conflates several effects of opposite sign: As a dominant firm becomes more dominant, its incentives to innovate increase. The same increase in firm dominance decreases the fringe firm’s incentive to catch up when its technology falls behind the dominant firm’s. In absolute terms, the encouragement effect is greater than the discouragement effect. However, along the steady state the fringe firm is the technology laggard more often than the dominant firm, which in turn implies that, overall, the effect of firm dominance is to decrease the average innovation rate.\textsuperscript{2} We refer to this as the \textit{shadow of Google} effect. It is related to a common complain in several high-tech industries:

In some niches of the software business, Google is casting the same sort of shadow over Silicon Valley that Microsoft once did. “You’ve got people who don’t even feel they can launch a product for fear that Google will get in.”\textsuperscript{3}

In the model’s context and absent technology transfer, “Google getting in” means imitation by a dominant firm. Even if the dominant firm does not reach the same level as the technology leader (and fringe firm), its complementary assets allow it to eat considerably into the fringe firm’s market share.

Matters change considerably when we consider markets for technology: in fact, technology transfer leads to an increase in the innovation rate; and an increase in market dominance leads to an increase in the innovation rate too. Suppose the fringe firm innovates. Such

\begin{itemize}
\item \textsuperscript{1} As in Aghion et al. (1997) and Segal and Whinston (2007), the technology state may be understood as the reduced form of a quality-ladder model with imitation (so that the technology laggard can always move one step behind the technology leader).
\item \textsuperscript{2} This is one of several instances where a dynamic model provides answers qualitatively different from, or unattainable by, a static game.
\item \textsuperscript{3} \textit{The New York Times}, May 2, 2006.
\end{itemize}
innovation is more valuable if in the hands of the dominant firm. Accordingly, Nash bargaining implies that the technology is transferred to the dominant firm at a price that splits the gains from an agreement. This in turn implies that the fringe firm partly internalizes the dominant firm’s value from innovation, leading to an increase in total gain from innovation. We refer to this as the innovation for buyout effect.

Now that Google has reportedly agreed to buy Israeli crowd-powered navigation app Waze for $1.3 billion, many other “Silicon Wadi” startups are daring to dream big.\(^4\)

In sum, the prospect of selling to a “giant” increases the payoff from innovation by a fringe firm. And a further increase in the size of the “giant” further increases the expected benefit for a fringe firm to innovate.

We then introduce an additional model variation, one that allows for radical innovation, defined as innovation that creates a new dominant firm (as opposed to incremental innovation, which leads to a higher technology level). Against the positive effect of technology transfer on incremental innovation, a third result states that technology transfer leads to a decrease in radical innovation. Intuitively, when there is technology transfer the fringe firm expects that, by succeeding in incremental innovation, it attains a higher payoff than without technology transfer. As a result, technology transfer increases the “opportunity cost” of radical innovation for the technology lagging fringe firm, which in turn reduces its radical innovation incentives. We refer to this as the complacency effect of technology transfer.

**Implications.** These results have several policy implications. First, regarding the issue of antitrust on innovation (Segal and Whinston, 2007), it’s important to distinguish radical innovation (competition for the market) from incremental innovation (competition within the market). For example, a soft antitrust policy toward dominant firms leads to an increase in radical innovation (partly Microsoft’s point in the *US v Microsoft* case) but also leads to a decrease in incremental innovation (partly the government’s point in the same case).

Second, the results have implications for the relation between two public policy instruments: the treatment of dominant firms and the protection of intellectual property (IP). A lenient treatment of dominant firms or strong patents are substitute instruments to increase the radical innovation rate. Intuitively, both increase the “prize” of being the dominant firm (and, most often, the technology leader as well). However, absent markets for technology a tough treatment of dominant firms or strong patents are substitute instruments to increase the incremental innovation rate.

Third, two qualifications must be made to the previous statement when considering the moderating effect of markets for technology. First, when it comes to the acquisition of fringe firms by dominant firms, a tougher treatment of dominant firms (a more restrictive merger policy) leads to lower incremental innovation rates and higher radical innovation rates (the opposite of the effect of policy regarding abuse of dominant position). Second, the effect of IP protection on innovation is mixed: by increasing the prize from patenting, it increases incremental innovation; but, by improving the market for technology, it reduces the rate of radical innovation.

Finally, with respect to the issue of persistence of leadership, we argue that the possibility of technology transfer separates the question of “who innovates” from the question of “who is the technology leader” (in the sense of owning the leading technology). The replacement effect (Arrow, 1962; Reinganum, 1983) implies that technology laggards are more likely to innovate; but the joint-profit effect (Gilbert and Newbery, 1982) implies that dominant firms are more likely to persist as technology leaders.

**Literature review.** The literature on innovation is fairly extensive. Reinganum (1989) provides an excellent survey of the work up to the 1980s, including several papers referenced below. One way to summarize this literature is to consider three main effects. First, the replacement effect, the idea that, by innovating, a technology leader cannibalizes its own profit stream, and hence has lower innovation incentives (Arrow, 1962; Reinganum, 1983; also related to Schumpeter, 1934). Second, the joint-profit effect (also known as the efficiency effect), the idea that a market leader has more to lose from not innovating than a challenger has to gain from innovating (Gilbert and Newbery, 1982; Budd et al., 1993; also related to Schumpeter, 1934).5 And third, the innovator-size effect (also known as the Arrow effect), the idea that the higher a firm’s output level, the greater the firm’s value of a quality increase or a cost decrease (Arrow, 1962; see also Schumpeter, 1942). The results in the present paper feature all of these three fundamental effects, but also other ones which are specific to the asymmetric nature of market structure considered explicitly in the paper.

Methodologically, the theoretical literature can be classified into three groups. First, one-race timing models such as Loury (1979), Lee and Wilde (1980) or Reinganum (1983). Second, one-race contest models such as Futia (1980) or Gilbert and Newbery (1982). Third, infinite contests (also know as ladder models) such as Harris and Vickers (1987), Aoki (1991), Budd, Harris and Vickers (1993) or Hörner (2004). The present paper is similar to the third group in that it considers an infinite contest; and it is related to the first group in that it develops a continuous-time model.

Of the more recent literature, Segal and Whinston (2007) is particularly germane. They “study the effects of antitrust policy in industries with continual innovation.” Specifically, they consider antitrust policy that changes the relative payoffs of technology leader and laggard. Similarly, this paper finds that “conflicting effects” are present in the comparative dynamics analysis of changes in α, the parameter that measures (inversely, in the present case) the intensity of antitrust policy. Two important differences of the present paper with respect to Segal and Whinston (2007) are that it (a) considers the possibility of firm acquisition, namely acquisition of a technology leader by a market leader; and (b) distinguishes between incremental and radical innovation.

Segal and Whinston (2007) assume that “if the potential entrant innovates, it receives a patent, enters, competes with the incumbent in the present period, and then becomes the incumbent in the next period, while the previous incumbent then becomes the potential entrant.” As such, their innovation combines incremental and radical innovation. This paper suggests that much of the confusion in the policy debate regarding dominant firms and innovation incentives stems from conflating incremental and radical innovation into one single dimension. One of the important results in this paper refers precisely to the trade-off between incremental and radical innovation (competition in the market and competition

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5. A variation of this effect is given by the escape-competition effect (Aghion et al., 1997). It is in turn related to the principle of least action from classical mechanics.
Aghion et al. (2005) “find strong evidence of an inverted-U relationship between product market competition and innovation.” To the extent that an increase in dominant firm’s dominance brings industry structure closer to the monopoly extreme, the results in this paper provide reasonable conditions under which market power diminishes the overall innovation rate, consistently with Aghion et al. (2005). However, the present paper also provides conditions under which the opposite is true.

The paper is also related to a recent literature focusing on technology transfer and markets for technology. Arora et al. (2001) and Gans and Stern (2003) identify the central drivers leading a start-up to either directly commercialize or sell its innovation. They show that one important condition is the efficiency of the “market for ideas.” By contrast, this paper considers the extreme cases when technology transfer is and is not possible. Gans and Stern (2000) analyze the relationship between incumbency and R&D incentives in a framework that combines elements of Gilbert and Newbery (1982) and Reinganum (1983). A key feature of their framework, which is ignored in the present paper, is the possibility of the incumbent threatening to engage in imitative R&D during negotiations for technology transfer (see also Gans et al., 2002). Spulber (2013) studies markets for technology. He argues that competitive pressures increase incentives to innovate. This is consistent with the result below regarding the effect of firm dominance on incremental innovation incentives. However, the present paper considers a world where technology transfer results from bilateral negotiation between innovators/competitors, whereas Spulber (2013) considers a market for inventions that brings together innovators and competitors.

Finally, a related strand of the literature deals with cumulative innovation, including Scotchmer (1991), Green and Scotchmer (1995), and Scotchmer (1996). These papers model cumulative innovation as a two-stage sequence of early and then later innovators. As such, this line of research does not take into account that the technology leaders of today may become technology laggards tomorrow.6

In sum, the above papers share some of the features of the framework developed in this paper. However, to the best of our knowledge, this is the first continuous time innovation model to address simultaneously the issues of firm dominance and technology transfer.7

**Roadmap.** The rest of the paper is organized as follows. Section 2 introduces the model, the basic assumptions, and the first set of results. Section 3 introduces the possibility of technology possible, and Section 4 the possibility of radical innovation. Section 5 discusses some implications of the results. Section 6 concludes the paper.

## 2. Basic model

Consider an industry with two firms in continuous time. There is a market dominant firm, denoted by the subscript \( M \); and a fringe firm, denoted by the subscript \( F \). At each moment, there is a technology leader, denoted by the subscript \( T \); and a technology laggard, denoted by the subscript \( L \).

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6. The first line in the paper’s title is motivated by Scotchmer’s (1991) use of Newton’s famous adage, as well as Audretsch’s variation applied to SBIR, a federal program designed to help small high-tech firms.

7. There are some additional related papers, including which will be referenced later in the paper.
Firms receive a flow of product market profits $\pi_{ik}$ and spend a flow of cost $C(x_{ik})$ to achieve an innovation hazard rate $x_{ik}$, all as a function of the firm’s state $ik$, where $i \in \{M, F\}$ and $k \in \{T, L\}$. At each moment, Nature determines the outcome of the innovation investments: innovation by firm $ik$ with a hazard rate $x_{ik}$. And if a technology laggard $iL$ innovates then it becomes the technology leader $iT$.

A note on terminology is in order: Until Section 4, we refer to innovation as the event that changes the technology state (a technology laggard becomes a technology leader). In Section 4, we introduce the possibility of radical innovation, the event that turns a fringe firm into a dominant firm. We then refer to technology-changing innovation as incremental innovation. However, for simplicity and until Section 4 “innovation” refers to incremental innovation.

The assumption that there are only two states regarding technology position — leader or laggard, $T$ or $L$ — echoes the assumption in Aghion et al. (1997) and Segal and Whinston (2007) that IP protection is limited and imitation is possible. This implies that, at all times, the technology laggard is able to remain at a given distance from the technology leader.

Let $V_{ik}$ be the value function of a firm in state $ik$. We have

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\begin{align*}
V_{ik} &= \int_0^\infty \exp(-(r + x_{ik} + x_{jl}) t) \left( \pi_{ik} - C(x_{ik}) + x_{ik} V_{iT} + x_{jl} V_{iL} \right) dt \\
&= \frac{\pi_{ik} - C(x_{ik}) + x_{ik} V_{iT} + x_{jl} V_{iL}}{r + x_{ik} + x_{jl}} 
\end{align*}
$$

where $i, j \in \{M, F\}$, $k, l \in \{T, L\}$, $i \neq j$, $k \neq l$. In words, firm $ik$ receives a payoff flow of $\pi_{ik} - C(x_{ik})$. This flow is interrupted when either firm $ik$ is successful in its innovation effort, leading to a switch to state $V_{iT}$; or when firm $jl$ (the rival firm) is successful in its innovation effort, leading to a switch to state $V_{iL}$. Finally, payoff flows are discounted at the combined rate $r + x_{ik} + x_{jl}$, which reflects both the passage of time and the likelihood of state-changing innovation events.

We make a set of assumptions that place some structure on the profit function and reflect the notions of market dominance and technology leadership. Let $\alpha$ be a parameter which measures the degree of market dominance. Specifically, suppose that the profit function $\pi_{ik}$ is parameterized by $\alpha$.

**Assumption 1.** (a) $\pi_{ik}$ (resp. $\pi_{Fk}$) is strictly increasing (resp. decreasing) in $\alpha$ ($k = T, L$); (b) $\pi_{ij}$ is strictly convex in $\alpha$ ($i = M, F; k = T, L$); (c) $\lim_{\alpha \to \infty} \pi_{Fk} = 0$ ($k = T, L$).

Part (a) corresponds to the assumption that $\alpha$ measures the degree of market dominance. Part (b) is a relatively standard IO assumption and is satisfied by a variety of specific models of product market competition. Part (c) is a limit condition stating that unbounded market dominance corresponds to de facto monopoly.

We also make some minimal and relatively standard assumptions regarding the innovation cost function.

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8. Goettler and Gordon (2011) develop a dynamic innovation model and estimate it with data from the personal computer microprocessor industry. A key distinctive feature of their model with respect to the present paper (and most of the innovation literature) is that they consider the implications of product durability for strategic decisions by buyers. By assuming state-dependent profit values $\pi_{ik}$ we effectively abstract from issues of durability and strategic buyers.
Assumption 2. $C(0) = C'(0) = 0$; if $x > 0$, then $C'(x) > 0$, $C''(x) > 0$, $C'''(x) > 0$

**Numerical computations.** In an effort to make the results as general as possible, no functional-form assumptions are made other than Assumptions 1–2. For the purpose of numerical computations that illustrate the theoretical results, we assume the innovation cost functions are quadratic. Regarding product market functions, the numerical computations are based on the following product market model. There exists a continuum of consumers (normalized to a measure 1), each of whom buys one unit from one of the two sellers. Specifically, each consumer receives net utility $u_{ik}$ from purchasing from a firm with market position $i$ ($i = M, F$) and technology position $k$ ($k = T, L$), which is given as follows:

$$u_{ik} = \alpha_i + \lambda_k - p_{ik} + \zeta_{ik}$$

where $\alpha_i$ denotes the extent of firm $ik$’s market leadership, $\lambda_k$ the extent of its technology leadership, $p_{ik}$ is firm $ik$’s price, and $\zeta_{ik}$ is the consumer’s utility shock from buying firm $ik$’s product. Specifically, $\alpha_i = \alpha$ if firm $ik$ is the dominant firm (that is, $i = M$), $\alpha_i = 0$ otherwise; and similarly, $\lambda_k = \lambda$ if firm $ik$ is the technology leader, $\lambda_k = 0$ otherwise.

Suppose $\zeta_{ik}$ is sufficiently large that the market is covered, that is, the outside option is always dominated by one of the sellers. Given that, we can work with $\xi_{ik} \equiv \zeta_{ik} - \zeta_{jl}$, the consumer’s relative preference for firm $ik$ ($j \in \{M, F\}$; $\ell \in \{T, L\}$; $j \neq i$; $\ell \neq k$). Assume also that $\xi_{ik}$ is distributed according to a normal $N(0, \sigma^2)$ distribution; and, with no further loss of generality, assume $\sigma^2 = 1$. It can be shown (Cabral and Riordan, 1994) that, under these assumptions,

$$\pi_{ik} = \frac{F(\lambda_i + \alpha_i)^2}{f(\lambda_i + \alpha_i)} \equiv \Pi(\lambda_i + \alpha_i)$$

where $F$ and $f$ are the cdf and pdf of $\xi$, respectively.

**First-order conditions.** The first-order conditions for value maximization are given by

$$\left(V_{iT} - C'(x_{ik})\right)(r + x_{ik} + x_{jl}) - \left(\pi_{ik} - C(x_{ik}) + x_{ik} V_{iT} + x_{jl} V_{iL}\right) = 0 \quad (2)$$

or simply

$$C'(x_{ik}) = V_{iT} - V_{ik} \quad (3)$$

Clearly, this leads to $x_{iT} = 0$, an implication of the well-known replacement effect: a technology leader does not benefit from innovation. We therefore focus on the values of $x_{iL}$, the innovation effort by technology laggards.

The first result states that, as firm asymmetry increases, the dominant firm has a greater incentive to innovate (when technology laggard), whereas the fringe firm has a lower incentive to innovate (when technology laggard).

9. Proposition 6 also assumes a quadratic cost function; this is the only exception to the rule that analytical results only require Assumptions 1 and 2.
10. The second-order conditions are given by

$$-C''(x_{i}) (r + x_{ik} + x_{jl}) + V_{iT} - C'(x_{ik}) - V_{iT} + C'(x_{ik}) = -C''(x_{i}) (r + x_{ik} + x_{jl}) < 0$$

where the last inequality follows from Assumption 2.
Figure 1
Numerical illustration of Proposition 1 \((r = \lambda = .1)\)

\[\text{Innovation rates } (x_{IL})\]

Asymmetry between dominant and fringe firm \((\alpha)\)

**Proposition 1.** \((a)\) \(x_{ML}\) is strictly increasing in \(\alpha\); \((b)\) \(x_{FL}\) is strictly decreasing in \(\alpha\)

The proof of this and the following results may be found in the Appendix. Proposition 1 is reminiscent of the innovator-size effect (Arrow, 1962): all things equal, larger firms have greater incentive to innovate than smaller firms. For example, a $1 cost reduction is worth more the greater the firm’s output. Therefore, as the degree of asymmetry increases, the dominant firm increases its innovation effort, whereas the fringe firm decreases its innovation (in both cases, when they are technology laggards). The reason why Proposition 1 is not as straightforward as the innovator-size effect is that the latter refers to an exogenously given profit function, whereas, in our model, the marginal incentive to innovate is given by the difference of endogenously determined value functions, as shown in (3).

Figure 1 illustrates Proposition 1. The horizontal axis corresponds to \(\alpha\), the measure of asymmetry between dominant and fringe firm. At \(\alpha = 0\), dominant and fringe firms choose the same level of innovation effort when technology laggards: \(\alpha = 0\) implies that the model is effectively symmetric. As \(\alpha\) increases, the innovation effort by the dominant firm, \(x_{ML}\), strictly increases, whereas the innovation effort by the fringe firm, \(x_{FL}\), strictly decreases.

Figure 1 also shows that the simple average of \(x_{ML}\) and \(x_{FL}\) is strictly increasing in \(\alpha\). This is reminiscent of the joint-profit effect (Gilbert and Newbery, 1982): in terms of profit change, the dominant firm has more to gain from an increase in \(\alpha\) than a fringe firm has to lose from a decrease in \(\alpha\). This basically results from the convexity of the profit function \(\pi\), which follows from Assumption 2 and is consistent with a number of oligopoly competition models (Gilbert and Newbery, 1982).

**Steady state.** At different times, the identity of the technology laggard is different, sometimes the dominant firm, sometimes the fringe firm. We are interested in the average innovation rate. Suppose that a visitor from Mars were to arrive in our planet; what innovation rate would it likely find? The answer to this question is given by the he steady-state innovation rate. Since the innovation rate is a hazard rate, the steady-state average
Figure 2
Numerical illustration of Proposition 2 ($r = \lambda = .1$)

![Figure 2](image)

Innovation rate ($X$)

Asymmetry between dominant and fringe firm ($\alpha$)

innovation rate is given by the harmonic mean of the innovation hazard rates:

$$X = \frac{\sum_{i \in \{M,F\}} \left( \frac{1}{x_{iL}} \left( \frac{1}{x_{ML}} + \frac{1}{x_{FL}} \right) \right) x_{iL}}{x_{ML} + x_{FL}}$$

Our next result shows that, on average, an increase in firm dominance leads to a decrease in innovation.

**Proposition 2.** There exists an $\alpha' > 0$ such that, if $\alpha > \alpha'$, then $X(\alpha) < X(0)$.

The intuition for Proposition 2 proceeds in three steps. First, by virtue of the replacement effect the relevant innovation incentives correspond to those of technology laggards. Second, as a dominant firm becomes more dominant, its incentives to innovate increase, whereas the fringe firm’s innovation incentives decline (Proposition 1). Third, in absolute terms, the encouragement effect is greater than the discouragement effect (cf Figure 1). Fourth, along the steady-state the probability that the dominant firm is the technology laggard decreases as $\alpha$ increases. In words, because of the previous effect (encouragement/discouragement effect), the weight placed on the encouragement effect decreases and the weight placed on the discouragement effect increases. In the limit when $\alpha \to \infty$, the fringe firm is almost always a technology laggard. Finally, the “intensive margin” and the “extensive margin” effects work in opposite ways in terms of the steady-state innovation probability. Proposition 2 shows that the “extensive margin” effect dominates, so that the innovation probability declines.

Figure 2 illustrates Proposition 2. As mentioned earlier, an increase in firm dominance (and increase in $\alpha$) implies an encouragement effect (higher effort when the dominant firm is a technology laggard, which corresponds to an increase in $x_{ML}$); but it also implies a discouragement effect (lower effort when the fringe firm is a technology laggard, which corresponds to a decrease in $x_{FL}$). If we give both effects equal weight, we get a positive change, as shown by the dashed line in Figure 2. However, as $\alpha$ increases the equilibrium weight placed on $x_{ML}$ decreases, whereas the equilibrium weight placed on $x_{FL}$ increases;
and the net effect is negative. The solid line in Figure 2 illustrates the end result: once we take into account the endogenous change in steady-state probabilities, the effect on $X$ of an increase in $\alpha$ is negative.

Another way to understand Proposition 2 is with reference to Equation (4). As can be seen, when computing the steady-state innovation probability $X$, the effect of the dominant firm’s effort ($x_{ML}$) is weighed by the fringe firm’s effort ($x_{FL}$). As the latter becomes smaller, the positive effect of the former becomes smaller. Similarly, the effect of the fringe firm’s effort ($x_{FL}$) is weighed by the dominant firm’s effort ($x_{ML}$). As the latter becomes bigger, the negative effect of the former becomes bigger.

To conclude this section, note that Proposition 2 provides an instance of when a dynamic model leads to a different result than a static model. Simply looking at the effect of market dominance on market profits, one may be tempted to suggest that innovation incentives increase with firm asymmetry (on account of the joint profit effect). However, a properly measured steady state effect shows that the opposite is true.

3. Technology transfer

A significant number of today’s most popular and successful products originated with smaller companies which were later gobbled up by one of the big players (Google, Microsoft, Yahoo, IBM, Oracle, etc). A very partial list includes Google acquiring Applied Semantics (Adsense), Android and YouTube; Microsoft acquiring Hotmail and Forethought (Powerpoint); and Facebook acquiring Instagram. These examples motivate a natural question: how are innovation incentives shaped by the possibility of innovator acquisition? And given that firm acquisition is possible, how does an increase in market dominance affect industry innovation incentives?

Assume that the moment we get to the ML state, the two firms transfer technology as the result of instantaneous Nash bargaining, so that we immediately move to state MT. The technology transfer price is given by $p$. Given this, at all times we are in the state where the dominant firm is a technology leader and the fringe firm a technology laggard. The value functions are now given by

$$V_{FL} = \frac{x_{FL}(V_{FL} + p) + \pi_{FL} - C(x_{FL})}{r + x_{FL}}$$

$$V_{MT} = \frac{x_{FL}(V_{MT} - p) + \pi_{MT}}{r + x_{FL}}$$

As before, the dominant firm makes zero investment in innovation when it is the technology leader (which happens at all time along the equilibrium path). Differently from before, innovation by a technology laggard fringe firm triggers the payment of price $p$ rather than the switch to the position of technology laggard.

11. We particularly interested in examining the effects of technology transfer on innovation incentives. For this reason, we consider a rather simple model of technology transfer. Hermalin (2013) models explicitly the relation between buyer and seller when there is asymmetric information and moral hazard. Spulber (2012), in turn, looks at the interaction of tacit knowledge with the trade-off between entrepreneurship and technology transfer. He shows that major inventions tend to result in entrepreneurship, minor inventions in technology transfer. This is consistent with my assumption that technology innovation only applies to incremental innovation.
The no-agreement subgame. In order to derive the (endogenously determined) technology transfer price $p$ we need to understand what the players’ outside options are with respect to a negotiated agreement. Accordingly, consider the (off-the-equilibrium-path) case when there is no agreement at $M\!L$. We assume this is a one-time disagreement, that is, it lasts until the next time $M$ innovates. (So, until then, $F$ does not innovate.) Denote by a superscript $\circ$ the off-the-equilibrium-path no-agreement value functions, that is, $V_M^{\circ}$ and $V_F^{\circ}$.

Firm $M$’s gains from an agreement are given by $V_M - p - V_M^{\circ}$, whereas Firm $F$’s gain are given by $V_F + p - V_F^{\circ}$. Nash bargaining predicts that the agreed-upon price is the value of $p$ that maximizes the product of the two gains. This implies

$$\hat{p} = \frac{1}{2} (V_M - V_M^{\circ}) + \frac{1}{2} (V_F^{\circ} - V_F)$$ (7)

The no-agreement value functions, in turn, are given by

$$V_M^{\circ} = \frac{x_M V_M + \pi_M - C(x_M)}{r + x_M}$$ (8)

$$V_F^{\circ} = \frac{x_M V_F + \pi_F}{r + x_M}$$ (9)

First-order conditions. From (8), the first-order condition for maximizing $V_M^{\circ}$ is given by

$$\left( V_M - C'(x_M) \right) (r + x_M) - \left( x_M V_M + \pi_M - C(x_M) \right) = 0$$

which in equilibrium implies

$$V_M - V_M^{\circ} = C'(x_M)$$ (10)

From (5), the first-order condition for maximizing $V_F$ is given by

$$\left( V_F + p - C'(x_F) \right) (r + x_F) - \left( x_F (V_F + p) + \pi_F - C(x_F) \right) = 0$$

In equilibrium, this implies

$$p = C'(x_F)$$ (11)

This equation clearly shows the nature of the game when there is technology transfer: the fringe firm follows a strategy of “innovation for buyout”: the benefit from innovation is simply given by the sale price $p$.

Proposition 3. In equilibrium, the technology transfer price is given by

$$p = \frac{(\pi_M - \pi_M^{\circ}) + (\pi_F - \pi_F^{\circ}) + C(x_F) + C(x_M)}{2 (r + x_M + x_F)}$$ (12)

Our goal is to determine innovation rates, not technology transfer prices. Proposition 3 is helpful for two reasons. First, Equation (12) is used as a building block of subsequent results regarding innovation rates. Second, it helps gain some intuition regarding the effects of technology transfer. Specifically, Nash bargaining implies that the price takes into account each party’s gain from an agreement. This is shown in Equation (12), where $\pi_M - \pi_M^{\circ}$.
and $\pi_{FT} - \pi_{FL}$ reflect (in part) the gain for the dominant firm and form the fringe firm, respectively. As shown before, the fringe’s firm first-order condition equates marginal cost to the sale price. Putting all of these together, we conclude that the effect of technology transfer is to “transfer” across firms some of the potential gains from an agreement. In particular, the fringe firm “internalizes” some of the dominant firm’s gain from an agreement.

Our next two results derive implications for the relation between firm dominance and the steady-state innovation rate $X$.

**Proposition 4.** There exists $\alpha' > 0$ such that, if $\alpha > \alpha'$ then the innovation rate is strictly increasing in $\alpha$ and strictly greater than under no technology transfer.

Technology transfer implies that the fringe firm partly internalizes the dominant firm’s gain from innovation; and the dominant firm’s gain from innovation is greater than the fringe firm’s gain; and the fringe firm’s incentive is what matters the most along the steady state. In other words, technology transfer turns the fringe firm’s strategy into one of innovation for buyout. This result is somewhat reminiscent of the joint-profit effect in innovation (Budd et al., 1993), with one important difference: in Budd et al. (1993), the leader has the greatest incentive to innovate; in our paper, the dominant firm has the greatest incentive to acquire the innovation.

Surprisingly, the condition in Proposition 4 that $\alpha$ be large is tight, as shown by the following result.

**Proposition 5.** There exists $\alpha' > 0$ such that, if $\alpha < \alpha'$ then the innovation rate is strictly decreasing in $\alpha$ and strictly lower than under no technology transfer.

In the neighborhood of $\alpha = 0$, the dominant firm and the fringe firms add to approximately a constant; and the rate at which $\pi_{Mk}$ varies is approximately the negative of the rate at which $\pi_{Fk}$ varies ($k = T, L$). In other words, the joint-profit effect, the main driver of the innovation for buyout effect, is of second order.

There is, however, a first-order effect: as $\alpha$ increases from $\alpha = 0$ the difference $\pi_{Mk} - \pi_{Fk}$, as well as its derivative with respect to $\alpha$, is positive and of first-order magnitude. This implies that the price received by the fringe firm declines: the total pie remains approximately constant, whereas the fringe firm’s slice strictly decreases. Finally, this implies a decrease in innovation incentives.

Figure 3 illustrates Propositions 4 and 5. The left panel shows that, for high values of $\alpha$, the innovation rate under technology transfer is increasing in $\alpha$ and greater than the innovation rate under no technology transfer. The right panel zooms in on low values of $\alpha$. Under no technology transfer, the innovation rate is flat at $\alpha = 0$: an infinitesimal increase in $\alpha$ leads to a symmetric “transfer” of innovation incentives from $F$ to $M$. Under technology transfer, however, firm $F$ (the one whose incentives are relevant) sees its gain from innovation strictly decrease as $\alpha$ increases. This results in an innovation rate that is decreasing and strictly lower than the innovation rate under no technology transfer.

In a related paper, Rasmusen (1988) shows that the possibility of buyout can make entry profitable which otherwise would not be. In other words, the possibility of firm acquisition increases entry incentives. Similarly, Proposition 4 implies that the possibility of firm acquisition increases incremental innovation incentives. Phillips and Zhidanov (2013), in turn, “show theoretically and empirically how mergers can stimulate R&D activity of small firms.” Although the context of their model is different, Proposition 4 is consistent with their theoretical and empirical results. In fact, mergers and acquisitions can be a form of technology transfer.
4. Radical innovation

Up to now we have assumed that the identity of the dominant firm is fixed. But dominant firms are not forever: over the years, markets have been reinvented with the emergence of new dominant firms. In this section we consider the possibility of radical innovation whereby the fringe firm becomes the next dominant firm. Specifically, by investing \( D(y_{ik}) \) firm \( ik \) becomes a dominant firm with hazard rate \( y_{ik} \).

In this section we specialize the analysis to the case of quadratic cost functions. Specifically, we assume that the cost of incremental innovation is given by \( C(x) = \frac{1}{2} x^2 \), where \( x \) is the rate of incremental innovation; whereas the cost of radical innovation is given by \( D(y) = \frac{1}{2\gamma} y^2 \), where \( y \) is the rate of radical innovation and \( \gamma \) a scaling parameter.

Consistently with the observation that radical innovation is rare, we consider the case when the value of \( \gamma \) is infinitesimal, so that the equilibrium radical innovation rate is also infinitesimal. This assumption has the advantage that, by continuity, the equilibrium rates of incremental innovation remain as before, that is, at the equilibrium levels absent radical innovation.

Our main result in this section pertains to the rate of radical innovation in relation to the degree of market dominance and the possibility of technology transfer.

**Proposition 6.** There exists \( \alpha', \gamma', r' > 0 \) such that, if \( \gamma < \gamma' \), \( \alpha > \alpha' \), and \( r < r' \), then the rate of radical innovation under no technology transfer is increasing in \( \alpha \) and strictly greater than under technology transfer.

First notice that, by the replacement effect, the dominant firm has no incentive for radical innovation. We thus focus on the fringe firm’s incentive. When there is technology transfer the fringe firm expects that, by succeeding in incremental innovation, it will attain a higher payoff than when there is no technology transfer. As a result, technology transfer increases the “opportunity cost” of radical innovation for the technology lagging fringe firm, which in turn leads to a lower incentive for radical innovation. We refer to this as the complacency effect of technology transfer: the ability to innovate for buyout makes life too good for a fringe firm to desire to switch places with the dominant firm. The first part of the result — that the radical innovation rate is increasing in \( \alpha \) — corresponds to the more straightforward
prize effect: a higher $\alpha$ implies a bigger prize from being a dominant firm, thus to a higher equilibrium radical innovation rate.

To conclude this section, we should note that the conditions in Proposition 6. Numerical simulations show that there are cases with either very low values of $\alpha$ or very high values of $r$ when incentives for radical innovation are higher under technology transfer. In this sense, Proposition 6 is more of a possibility result than the previous propositions regarding incremental innovation.

5. Discussion

This section discusses various implications of the results presented in the previous sections.

Policy towards dominant firms. A central focus of the present paper is on the effects of antitrust on innovation. Introducing this topic, Segal and Whinston (2007) state that, in the Microsoft case,

Microsoft argued that while as a technological leader it may possess a good deal of static market power, this is merely the fuel for stimulating dynamic R&D competition, a process that it argued works well in the software industry. Antitrust intervention would run the risk of reducing the rate of innovation and welfare. The government argued, instead, that Microsoft’s practices prevented entry of new firms and products, and therefore both raised prices and retarded innovation.

The results in this paper are consistent with Microsoft’s view — if we consider “dynamic R&D competition” from the radical innovation point of view. In fact, as Figure ?? shows, an increase in $\alpha$ (lenient antitrust policies) leads to an increase in radical innovation. However, Proposition 2 is consistent with the government’s view that Microsoft’s dominance (high $\alpha$) dampens incremental innovation, especially by fringe firms. This is then one of the main points that follow from the present analysis: it makes a big difference whether one refers to incremental innovation (competition within the market) or to radical innovation (competition for the market).

IP and antitrust policy instruments. At the risk of oversimplifying the policy debate, one may say that competition policy (or antitrust) is primarily based on instruments such as horizontal agreements and treatment of dominant firms; whereas innovation policy is primarily based on intellectual property (IP) protection instruments. This is unfortunate, for the various instruments are clearly related: IP protection has market power implications and the treatment of dominant firms has implications for innovation.

In the present context, the treatment of dominant firms is parameterized by $\alpha$: a higher value of $\alpha$ corresponds to a system that is more lenient towards dominant firms.\(^\text{13}\) Although the paper does not focus on IP policy, there is a natural parameter (used in the numerical computations) that reflects IP policy: $\lambda$. Recall that $\lambda$ measures the benefit from technology leadership; a stronger IP protection policy therefore corresponds to a higher value of $\lambda$.

\(^{13}\) There may be other factors that influence the value of $\alpha$, but here we focus on public policy toward dominant firms.
Figure 4 shows the steady-state rates of incremental and radical innovation as a function of $\alpha$ for two different levels of $\lambda$: $\lambda = .1$ (the base case considered throughout the paper) and $\lambda = .2$. As can be seen, a higher value of $\lambda$ leads to higher rates of incremental innovation, both with and without technology transfer. This is not surprising: it corresponds to the well-known “prize effect” of patents: the promise of rents leads firms to invest more.

Other than this uniform increase in innovation rates, an increase in $\lambda$ does not change the basic trade-offs regarding the value of $\alpha$ or the availability of markets for technology. Given this, we may say that, when technology transfer is possible, a lenient treatment of dominant firms (high $\alpha$) or strong patents (high $\lambda$) are substitute instruments to increase the (incremental) innovation rate. However, if technology transfer is not possible, then a tough treatment of dominant firms (low $\alpha$) or strong patents (high $\lambda$) are substitute instruments to increase the (incremental) innovation rate.

**Merger policy.** The results in this paper suggest that, in industries with dominant and fringe firms, an improvement in markets for technology leads to an increase in incremental innovation but also a decrease in radical innovation. There are various factors that contribute to better working markets for technology: one is the ability of dominant firms to acquire fringe firms.

This suggests some qualifications to the previous statements regarding antitrust and innovation policy. A tough treatment of dominant firms, as measured by $\alpha$, leads to higher incremental innovation rates and lower radical innovation rates. Examples of policy-induced changes in $\alpha$ include policies with respect to abuse of dominant position (as in the *US v Microsoft* or *DG Comp v Intel* cases). By contrast, when it comes to the treatment of dominant firms’ acquisition of fringe firms my results suggest that a tough treatment of dominant firms leads to lower incremental innovation and higher radical innovation rates.

With respect to innovation policy, stronger IP protection unambiguously contributes to higher incremental innovation rates, for two reasons: first, stronger patents increase the prize from successful innovation; and second, stronger IP rights contribute to better markets for technology, which in turn increases the incentives for incremental innovation. However,
the effect on radical innovation is ambiguous: on the one hand, the prize effect leads to higher innovation rates; on the other hand, better markets for technology make firms more complacent with the current dominant/fringe state, that is, less prone to engage in radical innovation.

**Strong property rights may destroy innovation incentives.** It is generally accepted that stronger property rights lead to more innovation. One possible exception to this principle stems from cumulative innovation (Scotchmer, 1991; Green and Scotchmer, 1995; Scotchmer, 1996). (In this regard, Galasso and Schankerman (2015) provide interesting empirical evidence.) The present paper suggests an additional reason why stronger property rights might have a counterproductive effect on innovation: to the extent that stronger IP rights lead to more efficient markets for technology, we may observe a decrease in radical innovation rates as suggested by Proposition 6.

**The incremental-radical innovation tradeoff.** Related to the previous point, Propositions 4 and 6 suggest a fundamental tradeoff: better markets for technology imply a higher rate of incremental innovation but a lower rate of radical innovation.

**Leadership persistence.** One of the central issues in the innovation literature is the degree to which leaders tend to remain as leaders, as opposed to being replaced by catching-up or leap-froging laggards. Arrow (1962) and Reinganum (1983) emphasize the importance of the replacement effect: to the extent that technology leaders would be cannibalizing their own product by producing a new one, laggards are more likely to innovate than leaders. The model in the present paper is consistent with this view: the technology leader’s innovation rate is zero.

Gilbert and Newbery (1982) point to a different effect (sometimes referred to as the efficiency effect or the joint-profit effect). (See also Budd et al., 1993; Cabral and Riordan, 1994.) If a given innovation were to be appropriated by the dominant firm or by the fringe firm (e.g., sold at an auction), then the dominant would have more to lose from not acquiring that innovation than the fringe firm. As such, we would expect that the dominant firm would end up owning the innovation. The analysis in Section 3 is consistent with this view: if the fringe firm produces an innovation, then efficient bargaining implies that the innovation is transferred to the dominant firm, thus implying persistence of technology leadership.

To put it differently, the possibility of technology transfer separates the question of “who innovates” from the question of “who is the technology leader” (in the sense of owning the leading technology). The replacement effect implies that technology laggards are more likely to innovate; but the efficiency effect implies that dominant firms are more likely to persist as technology leaders.

**Welfare analysis.** The analysis in the paper is focused on innovation rates; there is no claim regarding the optimality of firm dominance in terms of consumer or total welfare. However, in industries where innovation plays an important role in determining welfare one would expect the above results to be of first-order importance, and an increase in innovation rates to be associated to an increase in welfare.
Robustness and extensions. This paper considers the simple case when there is one dominant firm and one fringe firm. One possible extension is to consider \( n \) fringe firms. This would add more realism to the analysis but would not change the qualitative nature of the results.

A second natural change in the model refers to the assumption of efficient Nash bargaining. Clearly, technology transfer is not always efficient. The real world is somewhere between the two extremes considered in the paper (no technology transfer and efficient technology transfer). In principle, one would expect the effects in the intermediate case to be themselves intermediate. In other words, the closer the real-world is to efficient bargaining the closer the effects are to the world of efficient bargaining.

One could also assume a split division different from the 50-50 implied by Nash bargaining. In principle, this should not affect the qualitative nature of the results. However, it’s important that the split not be a function of \( \alpha \). If it is, then the the comparative dynamics with respect to \( \alpha \) might change. In particular, the fraction of the dominant firm’s gain that is internalized by the fringe firm may decrease in \( \alpha \), as the terms of technology transfer become increasingly worse for the seller.

Finally, we made the assumption that firms can perfectly control how to invest in innovation: \( x \) in incremental innovation, \( y \) in radical innovation. This is a rather extreme assumption: often a project that was thought to be incremental turns out to be radical, and vice versa. For the purpose of the results in this paper, the important assumption is that firms have some control over the direction of their innovative activity, so that they can make the distinction between \( x \) and \( y \) even if with noise.

Related empirical evidence. There are a number of recent empirical papers featuring results that relate to the assumptions or the results in this paper. First, Moser and Wong (2016) look at the effects of Monsanto’s entry into the soy seed breeding market (which resulted from Monsanto’s acquisition of DeKalb Genetics from 1996–1998). Among other effects, Moser and Wong (2016) report a significant decline in the incumbents’ innovation rates following the arrival of “giant” Monsanto. For example, incumbents performed 81% fewer field trials per firm for soy compared with other crops. This decline more than offset the increase created by Monsanto’s increase. This is consistent with the shadow of Google effect characterized by Proposition 2.

Watzinger et al. (2016) examine the effects of the 1956 Consent Decree which allowed Bell to remain as a telecommunications monopolist but forced it to license its patents royalty-free. They show that follow-on innovation increased by 11% on average, an effect that was driven primarily by young and small firms outside the telecommunication industry (the industry where Bell was a dominant firm). This is consistent with the shadow of Google effect characterized by Proposition 2: fringe firms have little incentive to innovate if they face a dominant firm in the product market.

Galasso and Schankerman (2015) explore a natural experiment from the US judicial system: the Federal Circuit US Court of Appeal assigns judges to patent cases in a random fashion. Specifically, Galasso and Schankerman (2015) look at the effects of Court-sanctioned patent invalidation on subsequent innovation. They report a 50% increase in subsequent citations. Moreover, they show that this effect is entirely driven by invalidation of patents owned by large patentees. This result suggests that the efficiency of contractual arrangements for technology transfer depends on the asymmetry between buyer and seller.
This in turn casts a word of caution over our assumption that the technology transfer bargaining solution is independent of $\alpha$. As mentioned earlier, if the dominant firm’s market power is sufficiently increasing in $\alpha$, then the innovation for buyout effect may be reversed.

### 6. Conclusion

Sir Isaac Newton famously stated that, “if I have seen far, it is by standing on the shoulders of giants.” Many recent examples from high-tech industries suggest that the opposite may be true, that it’s a case of “giants standing on the shoulders of dwarfs.” This paper considered two versions of this phenomenon: imitation and acquisition. The first version takes place when small firms invent only to see their ideas copied by “giants” who leverage their market power to effectively appropriate the value generated by “dwarfs.” The second version takes place when small inventors (“dwarfs”) are gobbled up by dominant firms (“giants”).

The various results developed in this paper suggest a number of policy implications, including:

- A soft antitrust policy toward dominant firms leads to an increase in radical innovation but a decrease in incremental innovation.
- A merger policy that restricts the acquisition of fringe firms by dominant firms leads to lower incremental innovation rates and higher radical innovation rates.
- The effect of IP protection on innovation is mixed: by increasing the prize from patenting, it increases incremental innovation; but, by improving the market for technology, it reduces the rate of radical innovation.

The analysis in the previous sections also suggests two lines of follow-up research: First, a calibrated model where incremental and radical innovation rates contribute to an overall innovation rate. This would allow us to understand the effect of market dominance and technology transfer on innovation, especially when there is a trade-off between incremental and radical innovation. Second, a demand model with more structure so as to estimate the welfare effects of innovation (the fact that innovation rates increase does not necessarily imply that welfare increases).

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Appendix

**Proof of Proposition 1:** Substituting (3) for \( C'(x_{ik}) \) in (2), the equilibrium value function for firm \( ik \) may implicitly be re-written as

\[
x_{ik} (V_{iT} - V_{ik}) + x_{jl} (V_{iL} - V_{ik}) + \pi_{ik} - C(x_{ik}) - r V_{ik} = 0
\]

Specifically, considering the cases when \( (k = t, l = l) \) and \( (k = l, l = t) \) we have

\[
x_{jl} (V_{iL} - V_{iT}) + \pi_{iT} - r V_{iT} = 0 \tag{13}
\]

\[
x_{il} (V_{iT} - V_{iL}) + \pi_{iL} - C(x_{iL}) - r V_{iL} = 0 \tag{14}
\]

where we use the fact that \( x_{it} = 0 \). Subtracting the first equation from the second, we get

\[
(V_{iT} - V_{iL}) (r + x_{jl} + x_{il}) - (\pi_{iT} - \pi_{iL}) - C(x_{iL}) = 0
\]

Substituting the first-order condition (3), we get

\[
C'(x_{iL}) (r + x_{jl} + x_{il}) - C(x_{iL}) = \pi_{iT} - \pi_{iL} \tag{15}
\]

Define, for a generic variable \( z \), \( \dot{z} \equiv \partial z / \partial \alpha \). Differentiating (15) with respect to \( \alpha \), for \( i = m, f \), we get the following system of equations:

\[
\begin{bmatrix}
  a_{MM} & a_{MF} \\
  a_{FM} & a_{FF}
\end{bmatrix}
\begin{bmatrix}
  \dot{x}_{ML} \\
  \dot{x}_{FL}
\end{bmatrix}
=
\begin{bmatrix}
  \hat{\pi}_{MT} - \hat{\pi}_{ML} \\
  \hat{\pi}_{FT} - \hat{\pi}_{FL}
\end{bmatrix}
\]

where

\[
\begin{align*}
  a_{MM} & \equiv C''(x_{ML}) (r + x_{ML} + x_{FL}) \\
  a_{MF} & \equiv C'(x_{ML}) \\
  a_{FF} & \equiv C''(x_{FL}) (r + x_{ML} + x_{FL}) \\
  a_{FM} & \equiv C'(x_{FL})
\end{align*}
\]

The solution to the above system is given by

\[
\Delta \cdot 
\begin{bmatrix}
  \dot{x}_{ML} \\
  \dot{x}_{FL}
\end{bmatrix}
=
\begin{bmatrix}
  a_{FF} & -a_{MF} \\
  -a_{FM} & a_{MM}
\end{bmatrix}
\begin{bmatrix}
  \hat{\pi}_{MT} - \hat{\pi}_{ML} \\
  \hat{\pi}_{FT} - \hat{\pi}_{FL}
\end{bmatrix} \tag{16}
\]

where \( \Delta \equiv a_{MM} a_{FF} - a_{MF} a_{FM} \). Assumption 2 states that \( C(0) = 0 \) and that \( C'(x) \) is an increasing, convex function (specifically, \( C''(x) > 0 \) and \( C'''(x) > 0 \)). Together, these properties imply that \( x C''(x) > C'(x) \). It follows that

\[
\Delta = C''(x_{ML}) (r + x_{ML} + x_{FL}) C''(x_{FL}) (r + x_{ML} + x_{FL}) - C'(x_{ML}) C'(x_{FL}) > C''(x_{ML}) x_{ML} C''(x_{FL}) x_{FL} - C'(x_{ML}) C'(x_{FL}) > 0
\]

From (16) we get

\[
\Delta \cdot \dot{x}_{ML} = C''(x_{FL}) (r + x_{ML} + x_{FL}) (\hat{\pi}_{MT} - \hat{\pi}_{ML}) - C'(x_{ML}) (\hat{\pi}_{FT} - \hat{\pi}_{FL}) > 0
\]

\[
\Delta \cdot \dot{x}_{FL} = -C'(x_{FL}) (\hat{\pi}_{MT} - \hat{\pi}_{ML}) + C''(x_{ML}) (r + x_{ML} + x_{FL}) (\hat{\pi}_{FT} - \hat{\pi}_{FL}) < 0
\]
since Assumption 1 implies that $\dot{\pi}_{MT} - \dot{\pi}_{ML} > 0$ and $\dot{\pi}_{FT} - \dot{\pi}_{FL} < 0$; whereas Assumption 2 implies that $C''(x_{FL}) > 0$, $C''(x_{ML}) > 0$, $C'(x_{FL}) > 0$, and $C'(x_{ML}) > 0$. ■

**Proof of Proposition 2:** By Assumption 1, $\alpha \rightarrow \infty$ implies that $\dot{\pi}_{MT} - \dot{\pi}_{ML} \rightarrow 0$. From (15), we conclude that, as $\alpha \rightarrow \infty$,

$$C'(x_{FL}) \rightarrow C'(x_{FL})/(r + x_{FL} + x_{ML}) < C(x_{FL})/x_{FL} = \overline{C}(x_{FL})$$

which, by Assumption 2, implies that $x_{FL} \rightarrow 0$. The result then follows from (4). ■

**Proof of Proposition 3:** The value functions (5), (6), (8), and (9) may be re-written as

\[
(r + x_{FL}) V_{FL} = x_{FL} (V_{FL} + p) + \pi_{FL} - C(x_{FL}) \quad (17)
\]

\[
(r + x_{FL}) V_{MT} = x_{FL} (V_{MT} - p) + \pi_{MT} \quad (18)
\]

\[
(r + x_{ML}) V_{ML}^\circ = x_{ML} V_{MT} + \pi_{ML} - C(x_{ML}) \quad (19)
\]

\[
(r + x_{ML}) V_{FT}^\circ = x_{ML} V_{FL} + \pi_{FT} \quad (20)
\]

respectively. Subtracting the third equation from the second, and simplifying, we get

\[
(r + x_{ML})(V_{MT} - V_{ML}^\circ) = \pi_{MT} - \pi_{ML} + C(x_{ML}) - x_{FL} p \quad (21)
\]

Subtracting the first equation from the fourth, and simplifying, we get

\[
(r + x_{ML})(V_{FT}^\circ - V_{FL}) = \pi_{FT} - \pi_{FL} + C(x_{FL}) - x_{FL} p \quad (22)
\]

Adding (21) and (22); using (7); and simplifying, we get the expression in the proposition. ■

**Proof of Proposition 4:** The sign of $dx_{ML}/d\alpha$ is the same as the sign of the right-hand side of (26), that is,

\[
((2r + 2x_{ML} + x_{FL}) C''(x_{FL}) + C'(x_{FL})) (\dot{\pi}_{MT} - \dot{\pi}_{ML}) - (x_{FL} C''(x_{FL}) + C'(x_{FL})) (\dot{\pi}_{FT} - \dot{\pi}_{FL})
\]

By Assumption 1, $\dot{\pi}_{MT} - \dot{\pi}_{ML} > 0$ and $\dot{\pi}_{FT} - \dot{\pi}_{FL} < 0$. By Assumption 2, $C''(x_{FL}) > 0$ and $C'(x_{FL}) > 0$. It follows that $x_{ML}$ is strictly increasing in $\alpha$.

If $x_{FL} > x_{ML}$ and $x_{FL}$ is decreasing in $\alpha$, then there exists an $\alpha'$ such that $x_{FL} < x_{ML}$. We thus consider the set of value of $\alpha$ such that $x_{FL} < x_{ML}$.

From (21) and (22), $V_{MT} - V_{ML}^\circ > V_{FT}^\circ - V_{FL}$ if and only if

$$\pi_{MT} - \pi_{ML} + C(x_{ML}) > \pi_{FT} - \pi_{FL} + C(x_{FL})$$

Since $\pi_{MT} - \pi_{ML} > \pi_{FT} - \pi_{FL}$ by Assumption 1 and $C(x_{ML}) > C(x_{FL})$ by Assumption 2 and $x_{FL} < x_{ML}$, we conclude that $V_{MT} - V_{ML}^\circ > V_{FT}^\circ - V_{FL}$.

From (27), the sign of $dx_{FL}/d\alpha$ is the same as the sign of the right-hand side of (27), that is,

\[
\xi \equiv \left( (r + x_{ML}) C''(x_{ML}) - 2 (C'(x_{FL}) - C'(x_{ML})) \right) (\dot{\pi}_{MT} - \dot{\pi}_{ML}) + \left( (2r + 2x_{ML} + x_{FL}) C''(x_{FL}) + C'(x_{FL}) \right) (\dot{\pi}_{FT} - \dot{\pi}_{FL})
\]
By Assumption 1, $\pi_{FT} - \pi_{FL} \to 0$ as $\alpha \to \infty$. It follows that, as $\alpha \to \infty$,

$$\xi \to \left((r + x_{ML}) C''(x_{ML}) - 2(C'(x_{FL}) - C'(x_{ML}))\right)(\hat{\pi}_{MT} - \hat{\pi}_{ML})$$

$$> \left((r + x_{ML}) C''(x_{ML})\right)(\hat{\pi}_{MT} - \hat{\pi}_{ML}) > 0$$

where the first inequality follows from Assumption 2 and $x_{FL} < x_{ML}$, whereas the second inequality follows from Assumption 1 ($\hat{\pi}_{MT} - \hat{\pi}_{ML} > 0$) and Assumption 2 ($C''(x_{ML}) > 0$).

**Proof of Proposition 5:** Substituting (11) for $p$ and (10) for $V_{MT} - V_{ML}$ in (7), we get

$$V_{FT}^0 - V_{FL} = 2C'(x_{FL}) - C'(x_{ML})$$

(23)

Substituting (11) for $p$, (10) for $V_{MT} - V_{ML}$, and (23) for $V_{MT} - V_{ML}$ in (21)–(22), and simplifying, we get

$$(r + x_{ML}) C'(x_{ML}) + x_{FL} C'(x_{FL}) - C'(x_{ML}) = \pi_{MT} - \pi_{ML} \tag{24}$$

$$(r + x_{ML}) (2C'(x_{FL}) - C'(x_{ML})) + x_{FL} C'(x_{FL}) - C'(x_{FL}) = \pi_{FT} - \pi_{FL} \tag{25}$$

Equations (24) and (25) form a system of two equations with two unknowns, $x_{ML}$ and $x_{FL}$. Differentiating with respect to $\alpha$ and solving we get

$$\Delta \cdot \dot{x}_{ML} = \left((2r + 2x_{ML} + x_{FL}) C''(x_{ML}) + C'(x_{FL})\right)(\hat{\pi}_{MT} - \hat{\pi}_{ML}) -$$

$$-\left(x_{FL} C''(x_{FL}) + C'(x_{FL})\right)(\hat{\pi}_{FT} - \hat{\pi}_{FL}) \tag{26}$$

$$\Delta \cdot \dot{x}_{FL} = \left((r + x_{ML}) C''(x_{ML}) - 2(C'(x_{FL}) - C'(x_{ML}))\right)(\hat{\pi}_{MT} - \hat{\pi}_{ML}) +$$

$$+\left((2r + 2x_{ML} + x_{FL}) C''(x_{ML}) + C'(x_{FL})\right)(\hat{\pi}_{FT} - \hat{\pi}_{FL}) \tag{27}$$

At $\alpha = 0$, $x_{ML} = x_{FL} = x$ and $(\hat{\pi}_{MT} - \hat{\pi}_{ML}) = -(\hat{\pi}_{FT} - \hat{\pi}_{FL})$. It follows that form (27) that

$$\dot{x}_{FL} \propto \left((r + x) C''(x) - 2(C'(x) - C'(x))\right)(\hat{\pi}_{MT} - \hat{\pi}_{ML}) -$$

$$-\left((2r + 2x + x) C''(x) + C'(x)\right)(\hat{\pi}_{MT} - \hat{\pi}_{ML})$$

$$= -((r + 2x) C''(x) + C'(x))(\hat{\pi}_{MT} - \hat{\pi}_{ML})$$

which is negative by Assumptions 1 and 2.

At $\alpha = 0$, the innovation rate is the same with or without technology transfer. Moreover, as seen before, the derivative of innovation rate with respect to $\alpha$ under no technology transfer is zero at $\alpha = 0$. We thus conclude that, (a) at $\alpha = 0$, the innovation is the same with or without technology transfer; (b) the derivative of the innovation rate with respect to $\alpha$ at $\alpha = 0$ is zero under no technology transfer and negative under technology transfer. The result then follows by continuity.

**Proof of Proposition 6:** Consider the case when there is no technology transfer. From (1)

$$(r + x_{ik} + x_{jl}) V_{ik} = \pi_{ik} - C(x_{ik}) + x_{ik} V_{iT} + x_{jl} V_{iL}$$

(1)
Expanding this system and taking into account that \( x_{it} = 0 \), we get

\[
\begin{align*}
(r + x_{FL}) V_{MT} &= \pi_{MT} + x_{FL} V_{ML} \\
(r + x_{ML}) V_{ML} &= \pi_{ML} - C(x_{ML}) + x_{ML} V_{MT} \\
(r + x_{ML}) V_{FT} &= \pi_{FT} + x_{ML} V_{FL} \\
(r + x_{FL}) V_{FL} &= \pi_{FL} - C(x_{FL}) + x_{FL} V_{FT}
\end{align*}
\]

Solving the system we get

\[
\begin{align*}
V_{MT} &= \frac{x_{FL} (\pi_{ML} - C(x_{MT})) + (r + x_{ML}) \pi_{MT}}{r (r + x_{FL} + x_{ML})} \\
V_{ML} &= \frac{(r + x_{FL}) (\pi_{ML} - C(x_{ML})) + x_{ML} \pi_{MT}}{r (r + x_{FL} + x_{ML})} \\
V_{FT} &= \frac{x_{ML} (\pi_{FL} - C(x_{FL})) + (r + x_{FL}) \pi_{FT}}{r (r + x_{FL} + x_{ML})} \\
V_{FL} &= \frac{(r + x_{ML}) \pi_{FL} + x_{FL} \pi_{FT} - (r + x_{ML}) C(x_{FL})}{r (r + x_{FL} + x_{ML})}
\end{align*}
\]

As shown in the proof of Proposition 4, \( x_{FL} \to 0 \) as \( \alpha \to \infty \). This implies that the relevant radical innovation incentive corresponds to the fringe firm as a technology laggard. Specifically, the first-order condition for effort in radical innovation is given by

\[
y_{FL} = \gamma (V_{ML} - V_{FL})
\]

From the solution to the system of value functions,

\[
(V_{MT} - V_{FT}) = \frac{(r + x_{ML}) \pi_{MT} - (r + x_{FL}) \pi_{FT} + x_{FL} (\pi_{ML} - C(x_{ML})) - x_{ML} (\pi_{FL} - C(x_{FL}))}{r (r + x_{FL} + x_{ML})}
\]

Substituting \( x^2/2 \) for \( C(x) \); substituting 0 for \( x_{FL}, \pi_{LK}, \) and \( r \); and simplifying, we get

\[
r (V_{MT} - V_{FT}) = \pi_{MT}
\]

which is increasing in \( \alpha \) by Assumption 1.

Consider now the case of technology transfer. The fringe firm is always the technology laggard. It stands to gain \( V_{MT} - V_{FL} - p \) from radical innovation. The idea is that, as a result of radical innovation, the FL firm becomes ML; and, following the equilibrium strategies, it acquires the leading technology from its rival at price \( p \), thus becoming firm MT. It follows that

\[
y_{FL} = \gamma (V_{MT} - V_{FL} - p)
\]

From (5)–(6), we get

\[
\begin{align*}
(r + x_{FL}) (V_{MT} - V_{FL}) &= \left( x_{FL} (V_{MT} - p) + \pi_{MT} \right) - \left( x_{FL} (V_{FL} + p) + \pi_{FL} - C(x_{FL}) \right) \\
r (V_{MT} - V_{FL}) &= \pi_{MT} - \pi_{FL} + C(x_{FL}) - 2 x_{FL} p \\
r (V_{MT} - V_{FL} - p) &= \pi_{MT} - \pi_{FL} + C(x_{FL}) - (r + 2 x_{FL}) p
\end{align*}
\]
Substituting $x^2/2$ for $C(x)$; substituting 0 for $\pi_{LK}$, and $r$; and simplifying, we get

$$r \left( V_{MT} - V_{FL} - p \right) = \pi_{MT} + \frac{1}{2} x_{FL}^2 - (r + 2 x_{FL}) p$$

Making the same substitutions in the expression for $p$ in Proposition 3, we get

$$p = \frac{\left( \pi_{MT} - \pi_{ML} \right) + \frac{1}{2} x_{FL}^2 + \frac{1}{2} x_{ML}^2}{2 (x_{ML} + x_{FL})}$$

It follows that

$$r \left( V_{MT} - V_{FL} - p \right) = \frac{2 (x_{ML} + x_{FL}) \left( \pi_{MT} + \frac{1}{2} x_{FL}^2 \right) - 2 x_{FL} \left( \pi_{MT} - \pi_{ML} + \frac{1}{2} x_{FL}^2 + \frac{1}{2} x_{ML}^2 \right)}{2 (x_{ML} + x_{FL})}$$

$$= \frac{x_{ML} \pi_{MT} + x_{FL} \pi_{ML}}{x_{ML} + x_{FL}} + \frac{x_{ML} x_{FL} (x_{FL} - x_{ML})}{x_{ML} + x_{FL}}$$

$$< \pi_{MT} + \frac{x_{ML} x_{FL} (x_{FL} - x_{ML})}{x_{ML} + x_{FL}}$$

(28)

where the inequality follows from Assumption 2. From (10) and (11), $x_{FL} < x_{ML}$ if and only if $V_{MT} - V_{ML} > p$. From (7), this is equivalent to $V_{MT} - V_{ML} > V_{MT}^0 - V_{FL}^0$. From Assumption 1 and (21)-(22) we conclude that $x_{FL} < x_{ML}$. Finally, from (28) we conclude that

$$r \left( V_{MT} - V_{FL} - p \right) < \pi_{MT}$$

which implies that the rate of radical innovation is lower under technology transfer. 

References


