Rational Buyers Search When Prices Increase*

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Abstract. Motivated by observed patterns in business-to-business transactions, we develop a dynamic pricing model. Seller costs are perfectly correlated and evolve according to a Markov process. In every period, each buyer observes (for free) the price set by their current supplier, but not the other sellers' prices or the sellers' (common) cost level. By paying a cost s the buyer becomes "active" and benefits from (Bertrand) competition among sellers. We show that there exists a semi-separating equilibrium whereby sellers increase price immediately when costs increase but decrease price gradually when costs decrease. Moreover, buyers become active when prices increase but not otherwise. Although several other equilibrium models have explained asymmetric price adjustment, our differs in that it delivers a positive correlation between price increases and "search" intensity.

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* Some restrictions apply.

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1. Introduction

Many sellers routinely purchase inputs from regular suppliers. For example, Blinder et al. (1998) report that out of sample of GDP representative sellers, on average 85% of sales are to regular buyers. In this business-to-business (B2B) context, buyers are faced with a dilemma: either they do "business as usual" with their current supplier (that is, pay the quoted price); or, alternatively, they try to obtain a better deal. A better deal can be gotten in several ways, including negotiating with the current supplier or searching for better prices currently offered by rival suppliers. For example, typically automakers such as General Motors have long-term relationships with parts suppliers, but every so often solicit bids from outside suppliers as well.

In this paper, we develop a dynamic model of pricing and buyer "search" which is motivated by examples like the one above. Seller costs are perfectly correlated across sellers and evolve according to a Markov process. In each period, each buyer observes (for free) the price set by their current seller, but not the other sellers' prices or the sellers' (common) cost level. By paying a cost s, the buyer benefits from (Bertrand) competition among sellers (for example, the buyer convenes a second-price auction among suppliers).

Although we present our model as a search model, it differs from existing search models in the assumption that sellers can price discriminate between searchers and non-searchers. (In this sense, a more appropriate name for searchers might be "active" buyers.) We believe this assumption is quite reasonable in many B2B markets, as well as in some consumer markets such as cable TV services.[1]

Although parsimonious in its setup, our model produces rich dynamics. As in many infinite-period models, there exists a plethora of equilibria. We focus on equilibria that produce "pooling" or "separating" patterns in price dynamics.

First, we provide conditions for the existence of a pooling equilibrium: in it, price converges to a constant level and is invariant to cost shocks. This equilibrium provides an explanation for sticky prices that differs from the standard menu-costs explanation: in our equilibrium, sellers refrain from changing price for fear of "rocking the boat" and inducing buyers to search (which would reduce seller profit from such buyers).

Second, we provide conditions for the existence of a separating equilibrium: price varies between low and high levels as cost itself alternates between low and high levels. Although in this equilibrium prices are fully informative, social welfare is lower than in a pooling equilibrium. This is due to the fact that, unlike the pooling equilibrium, a separating equilibrium requires that search take place along the equilibrium path.

Finally, we provide conditions for the existence of a semi-separating equilibrium, an equilibrium that is separating for price increases and pooling for price decreases: when costs increase, sellers immediately increase price; and subsequently price decreases gradually, in a pattern that induces buyers not to search. This equilibrium fits two stylized facts from price and search dynamics: first, prices increase more rapidly than they decrease, a pattern sometimes known as "rockets and feathers" (e.g., Bacon (1991), Peltzman (2000), Lewis (2011)); and second, buyers search following large price increases (e.g., Ho, Hogan, and

1. One of this paper's co-authors recently called a wine club to cancel his subscription. Immediately, the operator said that there was an alternative, lower price which the customer could pay instead of the regular price. While this example does not exactly match our model's assumptions, it is suggestive of the idea of price discrimination between "passive" and "active" buyers, a central feature of our theory.

Morton (2014)).

We are by no means the first to provide a theoretical explanation for asymmetric price adjustment. Tappata (2009), for example, proposes a Varian (1980) type model of mixed-strategies and price dispersion. The model produces rockets-and-feathers asymmetric price adjustment. Intuitively, when costs are high, price dispersion is lower; lower price dispersion implies less buyer search; and less buyer search reduces the sellers' incentives to lower prices when costs decrease. Our story for asymmetric price adjustment is very different. We show that, starting from a low cost state, there is equilibrium separation (prices increase if and only if costs increase); whereas, starting from a high-cost state, there is equilibrium pooling (prices gradually decrease as the buyers' beliefs about costs change). Unlike Tappata (2009), our sellers play pure strategies; the price dispersion that induces buyer search results from the assumption that sellers can discriminate between active and passive buyers.

In contrast with Tappata (2009), Yang and Ye (2008) propose a Salop and Stiglitz (1977) type bargains-and-ripoffs model. Capacity constrained sellers mix between a high and a low price. Some buyers have very low search cost and always search; some buyers have very large cost and never search; and an intermediate set of "critical" buyers search with some probability. When prices increase, searchers learn that cost is high and will likely remain high for some time. As a result, they stop searching, which in turn implies a slow decline in prices.

Although Yang and Ye (2008) and Tappata (2009) differ in the details of seller pricing, they share a similar story for asymmetric price adjustment. They also share the same prediction regarding buyer behavior: there is more search when prices are low and less search when prices are high. By contrast, our semi-separating equilibrium features search when prices *increase*. As often is the case in industrial organization, the question is not so much which model is right as which model better fits each industry. Our assumption that sellers can discriminate between searchers and regulars makes our model better applicable to ongoing services such as cable TV or B2B customer markets than for repeated "anonymous" purchases such as gasoline.

To the best of our knowledge, the only dynamic model predicting that a price increase leads to search is Lewis (2011). He assumes that buyers form expectations about the price distribution based on the average price level from the previous period. In this context, when prices increase buyers expectations of the price distribution tend to be too low, causing them to search more than they otherwise would. Our paper differs from his in that we assume buyers are rational and hold correct beliefs regarding seller prices. In other words, we present a complete equilibrium story for the prediction that buyers search when prices increase.

The other dynamic equilibrium search model that we are aware of is by Cabral and Fishman (2012), who propose a dynamic version of a Diamond (1971) type of model. Unlike Cabral and Fishman (2012), we assume that sellers can discriminate with respect to searchers (that is, active buyers). As a result, our equilibrium does not suffer from the well-known Diamond paradox (that there is no search in equilibrium even if the search cost is arbitrarily small and sellers set monopoly prices). Similarly to Cabral and Fishman (2012), we obtain asymmetric price adjustments to costs changes, though for a different reason.

The literature on search and price dispersion extends well beyond the above papers, including Burdett and Judd (1983), Stahl (1989), Benabou and Gertner (1993), Janssen and Moraga-González (2004). These papers develop static models, which makes the comparison to ours difficult. In particular, these papers are silent with respect to the question of buyer behavior in reaction to a price change.

■ **Roadmap.** The rest of the paper is structured as follows. In Section 2, we set up the basic model components, which include a cost level that evolves according to a two-state Markov process. Section 3 deals with the particular cases when cost dynamics have an absorbing state (either the high-cost or the low-cost state). Section 4 deals with the case when costs follow a stationary stochastic process. The reason for Section 3 is twofold: First, much of the intuition for the general case can be obtained from the simpler cases. Second, some results from the simpler, absorbing-state cases will be used as building blocks in the derivation of the stationary-cost-process case.

We discuss possible extensions of the model in Section 5 and conclude in Section 6.

2. Model

Consider a discrete time, infinite period model with two sellers ($i = 1, 2$) and a measure m of buyers. Both sellers and buyers discount the future according to the factor δ. Sellers produce the same product and face the same unit cost, c. We assume $c \in \{c_L, c_H\}$ and that c follows a Markov process with transition matrix

$$M = \begin{bmatrix} 1 - \gamma_L & \gamma_L \\ \gamma_H & 1 - \gamma_H \end{bmatrix}$$

For $i \in \{L, H\}$, γ_i is the probability of a cost change when in state i. Each buyer has a per-period unit demand with choke price $\overline{u}$, which we assume is very large, so that all buyers make a purchase in equilibrium.

The timing within each period runs as follows. Sellers observe the value of c and simultaneously set prices p_i. Each buyer is assigned to the seller it purchased from in the previous period. (In the first period, buyers are randomly allocated across sellers.) Each buyer observes the price (but not the cost) of the seller they are assigned to and chooses between two options: (a) buy from the current seller at the going price (passive buyers); or (b) become active and search for a better deal by paying a cost s. We make the important assumptions that, by paying s, the buyer is perceived by both sellers as an active buyer (i.e., a searcher); and that sellers are able to set different prices to active buyers, prices that we denote by q_i. Finally, sellers simultaneously set prices for searchers; searchers make their choices of seller; searchers and non-searchers make their choices of whether to make a purchase; and period payoffs are received by sellers and buyers.

A strategy for seller i consists of prices $p_i(t), q_i(t)$ to set in each period, possibly as a function of a history $\{c(\tau), p_i(\tau), q_i(\tau)\}_{\tau=0}^{t-1}$ ($i = 1, 2$) of costs and prices. A strategy for a buyer attached to seller i consists of (a) a choice of whether to become active (by which we mean paying cost s to get quotes q_i); and (b) a decision of whether to purchase or not, given the available prices ($p_i(t)$ for a passive buyer, $\{p_i(t), q_1(t), q_2(t)\}$ for an active buyer); where both decisions (a) and (b) are a function of a history of past prices observed by the buyer, a subset of $\{p_i(\tau), q_i(\tau)\}_{\tau=0}^{t-1}$ ($i = 1, 2$), as well as a belief $\beta(t)$ regarding the current cost level, where $\beta(t)$ is the probability at time t that $c = c_H$. An equilibrium is a set of strategies for sellers and buyers, as well as a belief by buyers regarding seller cost such that

(a) no player can improve payoffs by unilaterally changing their strategy and (b) beliefs are consistent with strategies.

As frequently happens in games with infinite-period games, there are multiple history-dependent equilibria. We focus on equilibria where buyer strategies are only a function of current price levels, differences in price levels from the previous period, and their prior on the underlying cost. Even within this set, and is often the case in games with incomplete information, there exist multiple equilibria. We first consider a separating equilibrium, where seller prices are a function of cost; and then a pooling equilibrium, where seller prices are a function of time but not cost level.

For ease of notation, when there is no danger of confusion we drop the argument t from prices and cost levels. Specifically, in our separating equilibrium price is only a function of the cost state, not of time. We thus denote by p_L (resp. p_H) the price level when $c = c_L$ (resp. $c = c_H$); and so forth.

3. Absorbing-state cost dynamics

In this section, we consider the particular cases when cost dynamics have an absorbing state. First, we consider the increasing-cost case: $c(0) = c_L$ and $\gamma_H = 0$. In words, firm cost starts off at a low level and increases with probability γ_L; and $c = c_H$ is an absorbing state. Then we consider opposite case when c decreases to a low cost absorbing state. Finally, in Section 4 we consider the case when c follows a stationary process.

3.1. Separating equilibrium

We first consider the possibility of a (symmetric) separating equilibrium: sellers set p_L when $c = c_L$ and p_H when $c = c_H$; buyers, in turn, search with probability α when price switches from p_L to p_H, and do not search otherwise; and always make a purchase. Buyers who become active at time t pay price $q = q_L$ if $c = c_L$ and $q = q_H$ if $c = c_H$ (both sellers set the same price for active buyers, that is, $q_1 = q_2 = q_k$, where $k \in \{L, H\}$ as the case may be). Since we are considering a separating equilibrium, the information obtained by active buyers at time t has no value in future periods, so in the next period active buyers face the same problem as other passive buyers.

Let v_k be seller value per buyer attached to the seller, measured at the beginning of the period, when the cost state is k ($k \in \{L, H\}$) and the buyer is passive (which in equilibrium is true always except when cost switches from c_L to c_H).[2] When $c = c_H$, we have

$$v_H = \frac{p_H - c_H}{1 - \delta} \tag{1}$$

In state $c = c_H$ a buyer is indifferent between searching and not searching in a given period if and only if

$$p_H = q_H + s \tag{2}$$

In fact, after getting a special deal by searching, price reverts back to p_H.

2. Note that the values v_k typically depend on buyer beliefs about c. In a separating equilibrium buyers are fully informed, so the information structure is irrelevant; but this is, of course, an equilibrium outcome.

At this point, it may be worth recalling that our sense of search differs from the conventional sense. In a separating equilibrium, active buyers (searchers) do not learn anything about prices, that is, buyers hold precise estimates of each seller's price. The benefit form being an active buyer is then to "force" sellers to offer a better deal (q instead of p). Specifically, when competing for an active buyer, sellers lower prices to the point where discounted profit is zero (Bertrand competition). This implies:

$$q_H - c_H + \delta\, v_H = 0 \tag{3}$$

Together, (1)–(3) imply

$$\begin{aligned} p_H &= c_H + (1-\delta)\, s \\ q_H &= c_H - \delta\, s \end{aligned} \tag{4}$$

To understand the intuition for these values, suppose that $\delta = 0$. Then, for a seller, buyers have no future value and $q_H = c_H$, that is, competing for an active buyer is like static Bertrand competition, yielding price equal to cost. Knowing this, buyers are willing to accept a price of $c_H + s$, one that exactly makes them indifferent between being active and being passive. At the opposite extreme, if $\delta = 1$ then a buyer is a valuable asset if $p_H > c_H$. Therefore, it must be that Bertrand competition implies that $p_H = c_H$.

Substituting (4) for p_H in (1) we get

$$v_H = s$$

In words, given the buyer's ability to force sellers to compete head to head by paying a cost s, the value of s is also the measure of the rent that a seller earns from a loyal buyer.

Suppose that buyers search with probability α when price increases from p_L to p_H. Then, when $c = c_L$, value per buyer is given by

$$v_L = p_L - c_L + \delta \Big(\gamma_L\, (1-\alpha)\, v_H + (1-\gamma_L)\, v_L \Big) \tag{5}$$

(Recall that a buyer who becomes active is effectively a customer lost, revenue wise; in other words, the seller is indifferent between keeping and losing a buyer who is offered price q_k.) Similarly to the case when $c = c_H$, when $c = c_L$ a buyer is indifferent between searching and not searching in a given period if and only if

$$p_L = q_L + s \tag{6}$$

Moreover, when $c = c_L$ competition for searchers implies

$$q_L - c_L + \delta \Big(\gamma_L\, (1-\alpha)\, v_H + (1-\gamma_L)\, v_L \Big) = 0 \tag{7}$$

Equations (5)–(7) can be solved to obtain

$$\begin{aligned} p_L &= c_L + s \Big(1 - \delta \left(1 - \alpha\, \gamma_L \right) \Big) \\ q_L &= c_L - s\, \delta \left(1 - \alpha\, \gamma_L \right) \\ v_L &= s \end{aligned} \tag{8}$$

The intuition for $v_L = s$ is similar to the intuition for $v_H = s$. The intuition for p_L, q_L is also similar to the intuition for p_H, q_H: In the limit when $\delta = 0$, we obtain $p_L = c_L + s$ and $q_L = c_L$. In the opposite extreme, when $\delta = 1$, we get $p_L = c_L + s\,\alpha\,\gamma_L$ and $q_L = c_L - s\,(1 - \alpha\,\gamma_L)$. These latter expressions differ from their high-cost counterpart because, at state $c = c_L$, there is always the chance that cost changes and a buyer is lost to search (which, along the equilibrium path, happens with probability α when cost changes from c_L to c_H, which in turn happens with probability γ_L).

Next, we consider whether there may be profitable deviations from the above strategies. First note that, by construction, buyers do not have a profitable deviation: they are always indifferent between searching and not searching, between purchasing and not purchasing. The binding constraint is therefore that firms do not want to deviate.

A deviation for a c_L type would be to "masquerade" itself as a c_H type and raise price before cost increases. This increases markup ($p_H - c_L > p_L - c_L$), but also results in the loss of an α fraction of buyers.[3] This gives us a lower bound on the value of α required for the above strategies to be equilibrium strategies.

Conversely, a deviation for a c_H type would be to "masquerade" itself as a c_L type and keep price at p_L when cost increases to c_H. By doing so, the seller retains the α fraction of buyers who search when price increases, but at a cost of a lower markup ($p_L - c_H < p_H - c_H$). This gives us an upper bound on the value of α required for the above strategies to be equilibrium strategies.

The next result summarizes the above discussion regarding a separating equilibrium:

Proposition 1. *There exist bounds $\overline{\alpha}$ and $\underline{\alpha}$, $0 \leq \underline{\alpha} < \overline{\alpha} \leq 1$, such that, if $\alpha \in [\underline{\alpha}, \overline{\alpha}]$, then a separating equilibrium exists. Passive (resp. active) buyers pay p_k (resp. q_k) when $c = c_k$ ($k = L, H$), where the values of p_k and q_k are given by (4) and (8). Passive buyers' beliefs that $c = c_H$ are given by $\beta = 1$ if $p = p_H$ and $\beta = 0$ otherwise. Buyers become active with probability α in the first period that $p = p_H$.*

The proof of this and subsequent results may be found in Appendix C. As frequently happens, we have a continuum of separating equilibria. A natural criterion is to select the equilibrium corresponding to the lowest value of α. This is a Pareto optimal equilibrium, since it minimizes "search" costs along the equilibrium path. Finally, one possible criticism of the equilibrium we consider is that consumers play mixed strategies. However, in this context we can think of mixed-strategies as a reduced form of pure strategies with privately observed shocks, in the tradition of Harsanyi (1973). In Appendix A we deal with this in greater detail.

3.2. Pooling Equilibrium

We now consider the possibility of a pooling equilibrium, that is, an equilibrium where the price that a seller charges to passive buyers does not depend on the seller's cost. Rather, this price is such that buyers are indifferent between being active or passive; and buyers are passive along the equilibrium path (that is, they always pay the cost quoted by the "usual"

3. Note that some of these buyers may remain with the seller. However, to the extent that buyers become searchers, their net present value is zero — just as if they actually left the seller for the rival seller.

seller). Given this equilibrium strategy, at time t buyers hold a belief β that cost is high, where

$$\beta(t) = 1 - (1 - \gamma_L)^t \tag{9}$$

Although in equilibrium buyers are passive, the value of becoming active determines their indifference condition and thus the highest price that sellers can charge to prevent buyers from becoming active. Suppose that a buyer decides to become active and that $c = c_H$ in that period. Sellers compete for the buyer's business, and so the discounted payoff from serving an active buyer is zero. Since sellers must pay c_H in each period to serve the customer in the future, it follows that the net present value of all prices sold to an active buyer is equal to the net present value of c_H in each period. It follows that the buyer's expected discount value is given by

$$\frac{\overline{u} - c_H}{1 - \delta}$$

In other words, the buyer receives utility $\overline{u}$ and pays a sequence of prices which, in terms of net present value, is equivalent to c_H in each period. By observing q_H an active buyer learns that $c = c_H$. This implies that such a buyer is more "pessimistic" about the value of c than passive buyers (that is, the buyer's belief that $c = c_H$ is greater than $\beta(t)$). Since passive buyers are indifferent between being active or passive, it follows that an active buyer who finds that $c = c_H$ strictly prefers to remain passive in the future.

In equilibrium, buyers purchase one unit in each and every period. This is true both along the equilibrium path and along the deviation path where a buyer becomes active. This implies that buyer value at every subgame takes the form $\overline{u}/(1 - \delta)$ plus an expression that does not depend on $\overline{u}$. (This is immediately evident in the above expression for expected value conditional on being active and conditional on $c = c_H$.) For this reason and for simplicity, we will compute buyer value functions net of willingness to pay $\overline{u}$. Specifically, if a buyer becomes active and cost happens to be high then we have

$$u_H = \frac{-c_H}{1 - \delta} \tag{10}$$

Suppose instead that the buyer becomes active when $c = c_L$. Now the buyer knows with certainty that the state is c_L at $t > 0$, and thus is more optimistic about the value of c than passive buyers (both in the current and in future periods). This implies that a buyer who becomes active at time $t > 0$ continues to do so as long as such buyer doesn't discover that $c = c_H$. Given this behavior, sellers correctly anticipate that the value in period $t + 1$ of a buyer who becomes active in period $t > 0$ is zero. Finally, Bertrand competition implies that sellers offer to sell at $q_L = c_L$.

The buyer's discounted value (when $c = c_L$) can be computed recursively as follows:

$$u_L = -c_L + \delta \Big((1 - \gamma_L)\, u_L + \gamma_L\, u_H - s \Big)$$

In the current period (and after having paid the search cost s), the buyer pays c_L. Next period, the buyer is again active (thus paying s) and two things may happen: with probability $1 - \gamma_L$ cost remains at c_L, in which case the buyer's value remains at u_L; and with probability γ_L cost switches to c_H, in which case the buyer receives u_H, as given by (10). The above recursive equation can be solved to

$$u_L = \frac{-c_L + \delta\,(\gamma_L\, u_H - s)}{1 - (1 - \gamma_L)\,\delta} \tag{11}$$

Let $u(t)$ be the value of a passive buyer at time $t > 0$ along the equilibrium path. Since the buyer is indifferent between being active or passive, $u(t)$ can be computed recursively

$$u(t) = -p(t) + \delta\, u(t+1) \tag{12}$$

The buyer's indifference condition (between being active or passive) also implies that value equals the value of being active in the current period:

$$\begin{aligned} u(t) &= \big(1 - \beta(t)\big)\, u_L + \beta(t)\, u_H - s \\ &= u_L - s - \beta(t)\,\big(u_L - u_H\big) \end{aligned} \tag{13}$$

In other words, given buyer indifference between being active or passive, we can compute buyer value by computing value from being active. By being active, the buyer pays s and expects, in the best possible case, u_L (where we note that $u_L > u_H$). This explains the $u_L - s$ term in (13). To the extent that the buyer is more pessimistic about the cost state, that is, to the extent that $\beta(t)$ is higher, the expected value from being active is lower, at the rate $-(u_L - u_H) < 0$. This explains the term $-\beta(t)\,\big(u_L - u_H\big)$ in (13).

Substituting (13) for $u(t)$ and $u(t+1)$ in (12), we get

$$p(t) = (1-\delta)\,(s - u_L) + \Big(\beta(t) - \delta\,\beta(t+1)\Big)\,\big(u_L - u_H\big) \tag{14}$$

and substituting (10) and (11) for u_H and u_L in (14), we obtain equilibrium price at $t > 0$:

$$p(t) = c_H + (1-\delta)\, s - (1-\gamma_L)^t\,\big(c_H - c_L - \delta\, s\big) \tag{15}$$

Next, consider the prices set to active buyers. An active buyer who discovers that $c = c_L$ becomes more optimistic than the average buyer, and so remains active until it finds that $c = c_H$. It follows that, beginning in the next period, such buyer has zero value for the seller. Therefore, Bertrand competition implies that

$$q_L(t) = c_L \tag{16}$$

By contrast, an active buyer who discovers that $c = c_H$ when $t > 0$ becomes more pessimistic than the average buyer, and so remains passive in future periods. Bertrand competition implies that

$$q_H(t) - c_H + \sum_{\tau=1}^{\infty} \delta^\tau\,\big(p(t+\tau) - c_H\big) = 0$$

Substituting (15) for p and simplifying, we get

$$q_H(t) = c_H - \delta\, s + \frac{\delta\,(1-\gamma_L)^{t+1}}{1 - \delta\,(1-\gamma_L)}\,(c_H - c_L - \delta s) \tag{17}$$

The case $t = 0$ differs from $t > 0$ in an important way: If a buyer becomes active at $t = 0$, no information about cost is gained with respect to passive buyers. As a result, an active buyer holds the same beliefs at $t = 1$ as passive buyers; and, similarly to those buyers, remains passive. This implies that the buyer's value is as given by (11) with the exception that s is not included:

$$u(0) = \frac{-c_L + \delta\,(\gamma_L\, u_H)}{1 - (1-\gamma_L)\,\delta} \tag{18}$$

Substituting (10) and (18) for u_H and u_L in (14), we obtain equilibrium price at $t = 0$:

$$p(0) = c_L + s - \frac{\delta s}{1 - \delta(1 - \gamma_L)} \tag{19}$$

as well as

$$q_L(0) = c_L - \frac{\delta\, s}{1 - \delta\,(1 - \gamma_L)} \tag{20}$$

The next result summarizes the above characterization of the pooling equilibrium:

Proposition 2. *If $s \geq (1 - \gamma_L)(c_H - c_L)$, then there exists a pooling equilibrium: passive buyers pay $p(t)$, which is given by (15) for $t > 0$ and (19) for $t = 0$; if $c = c_H$, then active buyers pay $q_H(t)$, which is given by (17); and if $c = c_L$, then active buyers pay $q_L(t)$, which is given by (16) for $t > 0$ and (20) for $t = 0$. Along the equilibrium path, passive buyers believe that $c = c_H$ with probability $\beta(t)$ given by (9). If a price different from $p(t)$ is observed at time t, then $\beta(t) = 0$. Buyers remain passive along the equilibrium path.*

Notice that, while the equilibrium establishes the values of the prices set to active buyers, no buyer becomes active in equilibrium. Observable equilibrium prices are therefore given by $p(t)$.

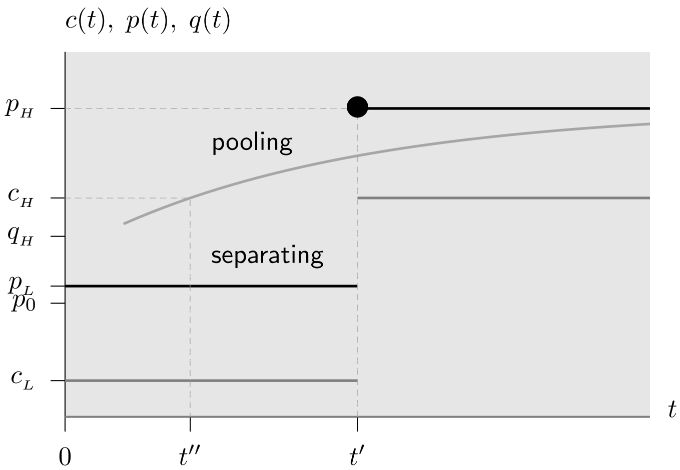
In deriving the pooling equilibrium we did not need to characterize the seller's value. However, an argument similar to the separating equilibrium shows that, at $t = 0$, the seller's value is simply given by s. In fact, a buyer who becomes active at $t = 0$ receives a discounted stream of prices equal to the discounted stream of costs (by virtue of Bertrand competition) and pays s. Since the buyer is indifferent between being active or passive at $t = 0$, it follows that, along the equilibrium path, the buyer pays a stream of prices equal to the discounted sum of costs. Finally, this implies that the seller's value is equal to s, the same value that the seller obtains under a separating equilibrium. In both cases, the argument is that the buyer is s dollars away from forcing sellers to bring discounted expected price down to discounted expected cost level, which implies that s is the measure of the rent earned by a seller from an attached buyer.

3.3. Comparing the pooling and separating equilibria

Figure 1 shows the values of p and q that are observed along the pooling and the separating equilibria, assuming that c switches from c_L to c_H at time t'. Consider first the separating equilibrium. While $c = c_L$ (that is, for $t < t'$), price (paid by non-searchers) is given by $p = p_L$ (black line). At $t = t'$, cost switches from c_L to c_H and price increases from p_L to p_H. A fraction α of buyers becomes active at this point and is offered price q_H. After than, all buyers remain passive and pay price p_H.

Contrast this to the pooling equilibrium, where buyers are always passive (along the equilibrium path) and the price paid by passive buyers is given by the gray line in Figure 1, for $t > 0$, whereas $p(0) = p_0$. For $t > 0$, equilibrium price in the pooling equilibrium falls between the extreme prices in the separating equilibrium: $p_L < p(t) < p_H$. In both equilibria, buyers are indifferent between being active or passive. Price level is then determined by buyer beliefs. In the separating equilibrium, beliefs switch from $\beta(t) = 0$ to $\beta(t) = 1$ as price increases from p_L to p_H. By contrast, in the pooling equilibrium beliefs gradually increase according to (9); and $0 < \beta(t) < 1$ implies $p_L < p(t) < p_H$.

Figure 1
Separating and pooling equilibria in increasing-cost case



At $t = 0$, however, we observe that equilibrium price in the pooling equilibrium is strictly lower than under the separating equilibrium: $p(0) < p_L$. At first, this appears to go against the characterization in the previous paragraph: at $t = 0$, buyers believe that $c = c_L$ (that is, $\beta(0) = 0$) in both equilibria. Why then isn't price the same under both equilibria? The answer is that, under the pooling equilibrium and when $t > 0$, sellers benefit from an asymmetric information rent as buyers do not know the value of cost (by contrast, along the separating equilibrium path buyers have the same information about costs as sellers). This implies that, if sellers compete for an active buyer at $t = 0$, they are willing to bring q down to a lower level; and this in turn implies that p must also be lower so as to keep buyers indifferent between being active or passive.

Why would a seller accept setting price below cost? Given the equilibrium strategies, the alternative of setting a higher price (e.g., a price above cost) leads all buyers to become active; and as we showed earlier an active buyer is worth zero. This implies that any upward deviation from the pooling equilibrium price implies a payoff of zero. Therefore, in order for the putative price path to be an equilibrium, we require that the seller receive positive profits. Although early on the seller may make negative period profits, its expect discounted profit is positive; in fact, it's positive with certainty, since after the cost switch there is no additional uncertainty. Naturally, the condition that discounted profit be positive implies limits on the relevant parameter values, which correspond to the condition in the text of Proposition 2.

3.4. The decreasing-cost case

Consider now the opposite absorbing-state dynamics case: $c(0) = c_H$ and $\gamma_L = 0$. In words, firm cost starts off at a high level and decreases with probability γ_H; and $c = c_L$ is an absorbing state. We call this the decreasing-cost case. Appendix B presents the parallel results to Propositions 1 and 2 for the decreasing-cost case. In what follows, we briefly describe the similarities and the differences with respect to the increasing-cost case.

As in the case where $\gamma_H = 0$, the separating equilibrium is characterized by prices p_L (p_H) and q_L (q_H) to be set when $c = c_L$ ($c = c_H$). Differently from the increasing-cost case, along the equilibrium path a positive measure of buyers must be active at all times when $p = p_H$. This follows from the seller's incentive compatibility constraint: if the seller does not lose any buyers by setting p_H, then it has no incentive to lower price to p_L when $c = c_L$ (as a separating equilibrium requires).

The pooling equilibrium is also constructed in a similar way to Section 3. As before, depending on the actual cost realizations, there may be periods when sellers price below cost. This imposes some restrictions on the parameter set so that the seller's participation constraint is satisfied.

4. The general case

We now come to the core of our paper, where we consider the more general case stationary stochastic process for cost: $\gamma_i \in (0,1)$, so that there are no absorbing states. The first part of the section follows a sequence similar to the previous section: we consider the possibility of a separating as well as a pooling equilibrium. After that, we consider the additional possibility of a hybrid equilibrium — a semi-separating equilibrium — which combine features of the separating and pooling equilibria.

4.1. Separating and pooling equilibria

Similarly to the equilibria in the previous section (absorbing-state case), we can construct a separating equilibrium such that in state $c = c_k$ ($k = L, H$) sellers charge non-searchers p_k and searchers q_k. The following result summarizes the main features of this separating equilibrium.

Proposition 3. *There exist bounds $\overline{\alpha}_L$, $\underline{\alpha}_L$, $\overline{\alpha}_H$ and $\underline{\alpha}_H$ such that, if $\alpha_L \in [\underline{\alpha}_L, \overline{\alpha}_L]$ and $\alpha_H \in [\underline{\alpha}_H, \overline{\alpha}_H]$, then a separating equilibrium exists. Passive (resp. active) buyers pay p_k (resp. q_k) when $c = c_k$, where the values of p_k and q_k are given by*

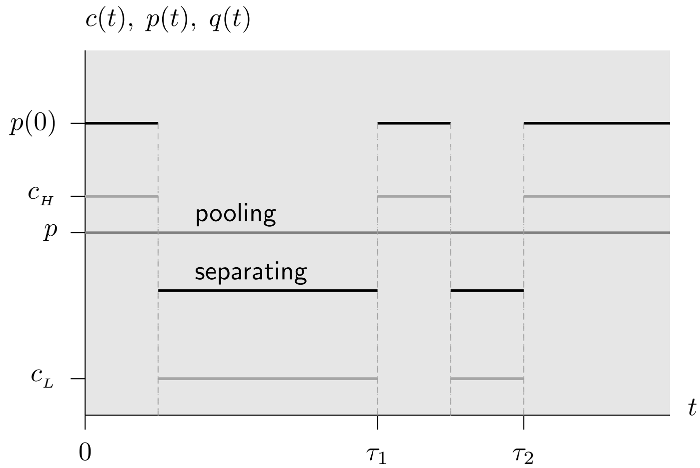
$$\begin{aligned} p_L &= c_L + (1-\delta)\, s + \delta\, \alpha_L\, \gamma_L\, s \\ q_L &= c_L - \delta\, s + \delta\, \alpha_L\, \gamma_L\, s \\ p_H &= c_H + (1-\delta)\, s + \delta\, \alpha_H\, s - \delta\, \alpha_H\, \gamma_H\, s \\ q_H &= c_H - \delta\, s + \delta\, \alpha_H\, s - \delta\, \alpha_H\, \gamma_H\, s \end{aligned}$$

Passive buyers' beliefs that $c = c_H$ are given by $\beta = 1$ if $p = p_H$ and $\beta = 0$ otherwise. Buyers are active with probability α_L in the first period that $p = p_H$ and with probability α_H in each period when the price remains at p_H.

We can also prove the existence of a pooling equilibrium. In this case, an additional parameter to consider is the initial belief regarding costs, β_0. In the previous section, we considered $\beta_0 = 0$ in the increasing-cost case and $\beta_0 = 1$ in the decreasing-cost case. In the stationary-cost case, a natural initial belief is that $c = c_H$ with probability equal to the steady-state average probability:

$$\beta_0 = \frac{\gamma_L}{\gamma_L + \gamma_H} \tag{21}$$

Figure 2
Separating and pooling equilibria in general case



For the purposes of stating the following result, we define

$$\overline{c}_k = \mathbb{E}\left(\sum_{t=0}^{\infty} \delta^t \, c_t \;\middle|\; c_0 = c_k \right)$$
$$\overline{c} = \frac{\gamma_L \, c_H + \gamma_H \, c_L}{\gamma_L + \gamma_H}$$

for $k = L, H$. In words, $\overline{c}_k$ is the expected discounted future stream of cost given that $c = c_k$ in the current period; and $\overline{c}$ is the unconditional steady-state expected value of cost.

Proposition 4. *Suppose that β_0 is given by (21). Then there exists a pooling equilibrium with a "sticky" price. Along the equilibrium path, buyers believe that $c = c_H$ with probability*

$$\beta(t) = \beta_0$$

Buyers remain passive along the equilibrium path and pay

$$p = (s - u_L)\,(1-\delta) + \beta_0\,(1-\delta)\,(u_L - u_H)$$

where

$$u_H = -\overline{c}_H$$
$$u_L = -\overline{c}_L - \frac{\delta}{1-\delta\,(1-\gamma_L)}\, s$$

If a price different from p is observed at time t, then $\beta(t) = 0$ and buyers become active. Active buyers pay

$$q_H = \overline{c}_H - \frac{\delta}{1-\delta}\, p$$
$$q_L = c_L$$

Figure 2 depicts the separating and the pooling equilibrium for a possible set of parameter values. As can be seen, under the pooling equilibrium there may be periods when price falls below cost. The reason why a seller may be willing to sustain (temporary) losses is that increasing price leads buyers to become active, which leads to Bertrand competition and zero value; by contrast, keeping price fixed keeps the buyer attached to the seller, with the associated promise of positive margins in the future when cost drops to c_L.

Although the initial belief (21) seems reasonable for a steady-state equilibrium, different values of β_0 also determine uniquely a pooling equilibrium. If $\beta(0) > \beta_0$, then the pooling equilibrium price drops gradually towards the equilibrium value p in Proposition 4; conversely, if $\beta(0) > \beta_0$, then the pooling equilibrium price increases gradually towards the equilibrium value p in Proposition 4.

4.2. Semi-separating equilibria

In addition to separating and pooling equilibria, we can construct semi-separating equilibria that combine features of the separating and pooling equilibria considered in the previous subsection. Specifically, suppose that cost changes from c_L to c_H at times τ_i, $i = 1, 2, ...$ A "natural" semi-separating equilibrium is as follows:

- Each time cost changes from c_L to c_H price increases and a fraction of buyers become active (as in the separating equilibrium)
- In subsequent periods, prices decline gradually and all buyers are passive (as in the cost-decreasing pooling equilibrium considered before).
- Time is "reset" each time cost shifts from c_L to c_H, so that strategies can be defined with reference to t, the number of periods elapsed since the last cost shift.

In this equilibrium, the seller's strategy is to set $p(t)$, where t is the number of periods since the last shift from c_L to c_H. The buyer's strategy is to be active at $t = 0$ with probability $\alpha(\tau_i - \tau_{i-1})$; and passive otherwise. (Notice that, differently from the separating equilibrium considered before, the fraction of buyers who become active when prices increase depends on how long it's been since the last price increase.)

Consider now buyer beliefs $\beta(t)$, where t is the number of periods elapsed since since the last cost shift. When cost switches from c_L to c_H (and so does price), buyers correctly believe that cost is high, that is $\beta(0) = 1$. In subsequent periods and until the next price increase buyers believe that cost has not decreased and increased again, that is, buyers believe that either cost has not changed or that it has shifted from c_H to c_L. It follows that

$$\beta(t) = (1 - \gamma_H)^t, \quad t = 0, 1, ..., \tau_{i+1} - \tau_i - 1$$

The buyer's indifference condition (between being active and being passive) is given by

$$\begin{aligned} &\beta(t)\, u_H + \big(1 - \beta(t)\big)\, u_L - s = \\ &\qquad = -p(t) + \delta \left(\mathbb{E}_t\left[\beta(t+1)\right]\, u_H + \big(1 - \mathbb{E}_t\left[\beta(t+1)\right]\big)\, u_L - s \right) \end{aligned}$$

The left-hand side gives expected price from becoming an active buyer in the current period; the right-hand side, expected price from being a passive buyer in the current period and becoming active in the next period. Solving with respect to p,

$$p(t) = c_L + s + (1-\gamma_H)^t \left(c_H - c_L - \delta\, \frac{1-\delta\,(1-\gamma_H)}{1-\delta}\, s \right) + \frac{\delta \left(1 - (1-\gamma_H)^t\right) \gamma_L}{1 - \delta\,(1-\gamma_H-\gamma_L)}\,(c_H - c_L) \tag{22}$$

Let $v_H(t)$ (resp. $v_L(t)$) be seller value per unit mass of buyers when the cost state is high (resp. low) and t periods have elapsed since the last cost shift. (Note that $v_L(0)$ is never realized, since $t=0$ is defined asa period when $c = c_H$; however, this value will be important for computing incentive compatibility conditions.) In equilibrium, these value functions satisfy

$$v_H(t) = p(t) - c_H + \delta\big((1-\gamma_H)\, v_H(t+1) + \gamma_H\, v_L(t+1)\big) \tag{23}$$

$$v_L(t) = p(t) - c_L + \delta\Big((1-\gamma_L)\, v_L(t+1) + \gamma_L\, \big(1 - \alpha(t+1)\big)\, v_H(0)\Big) \tag{24}$$

Specifically, suppose that t periods have elapsed since the last cost increase and suppose that cost is still at c_H. In the current period, the seller sets $p(t)$ and pays a cost c_H. Assuming the seller has a unit mass of customers, its current profit is given by $p(t) - c_H$. Beginning in the next period, two things may happen: cost remains high, yielding a continuation payoff $v_H(t+1)$; or cost drops to c_L, yielding a continuation payoff $v_L(t+1)$.

Now suppose that current cost is c_L. By a similar argument as in the preceding paragraph, current profit is $p(t) - c_L$. Beginning in the next period, two things may happen: cost remains low, yielding a continuation payoff $v_L(t+1)$; or cost increases to c_H. The latter outcome leads to a fraction $\alpha(t+1)$ buyers becoming active (which implies zero value for the seller) and a continuation value $v_H(0)$ times the measure of passive buyers.

Next we consider the seller incentive compatibility (IC) constraints. First, a seller may increase price when cost is low. This implies the following IC constraint:

$$v_L(t) \ge \big(1 - \alpha(t)\big)\; v_L(0) \tag{25}$$

Second, a seller may increase price when cost is (still) high. This implies the following IC constraint:

$$v_H(t) \ge \big(1 - \alpha(t)\big)\; v_H(0) \tag{26}$$

Finally, in a period when cost shifts from c_L to c_H the seller might deviate by *not* increasing price, rather continuing with the value $p(t)$ corresponding to no cost increase This implies the following IC constraint:

$$v_H(t) \le \big(1 - \alpha(t)\big)\; v_H(0) \tag{27}$$

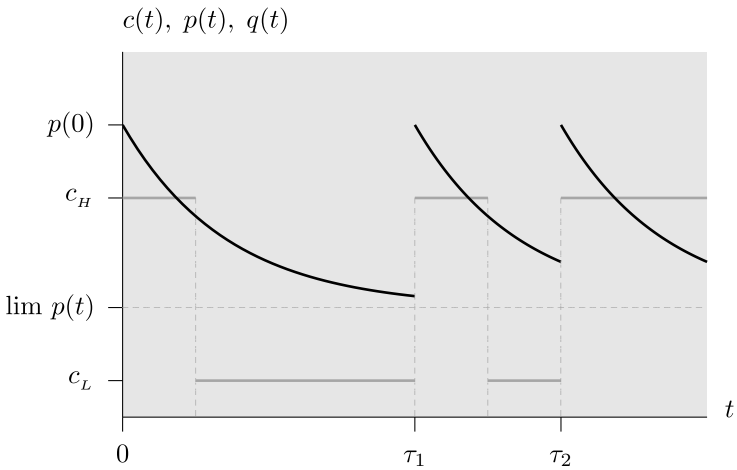
(26) implies (25). Moreover, (26) and (27) uniquely pin down the value of $\alpha(t)$:

$$\alpha(t) = 1 - \frac{v_H(t)}{v_H(0)} \tag{28}$$

We cannot find a closed-form solution to the value functions. However, we can derive them numerically and recursively. Since $p(t) \to \overline{p}$, so $v_i(t) \to \overline{v}$. These v_i limits can be solve from

$$\overline{v}_H = p(T) - c_H + \delta\big((1-\gamma_H)\,\overline{v}_H + \gamma_H\,\overline{v}_L\big)$$
$$\overline{v}_L = p(T) - c_L + \delta\big((1-\gamma_L)\,\overline{v}_L + \gamma_L\,\overline{v}_H\big)$$

Figure 3
Semi Separating equilibrium in dynamic case



The values of $p(t)$ and $v_i(t)$ can be arbitrarily approximated by considering an arbitrarily large terminal T, assuming $v_i(T+1) = \overline{v}_i$, and solving recursively.

Figure 3 depicts this semi-separating equilibrum for a particular set of parameter values and for a particular draw of the cost stochastic process. At times τ_1, τ_2, etc, cost shifts from c_L to c_H. When that happens, price immediately increases to $p(0)$ and a fraction α_i of the buyers becomes active. Between τ_i and τ_{i+1} prices decline gradually and all buyers are passive.

Finally, an alternative semi-separating has prices moving up as in a pooling equilibrium and downward as in a separating downward. In this equilibrium there must be active buyers in every period, although the probability of becoming active may vary. We conjecture that this equilibrium would lead to the worst level social welfare as "search" costs will be paid in every period.

5. Discussion

In this section, we discuss some of the more salient features of the separating, pooling and semi-separating equilibria developed in the previous sections.

■ **Price changes and "search."** One of the central features of the equilibria that we find is that buyers remain passive following a price decrease. In particular, the semi-separating equilibrium we derived in the previous section has the feature that buyers become active at time t if and only if the list price p increased with respect to period $t-1$. Define average price

$$r(t) \equiv \big(1 - \alpha(t)\big)\; p(t) + \alpha(t)\, q(t)$$

where $\alpha(t)$ is the measure of of active buyers in period t and $p(t), q(t)$ the list price and discounted price charged in period t. Depending on parameter values, it is possible that from τ_i to $\tau_i + 1$, r increases even though p decreases. Even so, it is the case that if there is "search" (active buyers) at time t then $r(t)$ is greater than $r(t-1)$. This prediction is in stark contrast with dynamic models of search — such as Tappata (2009) and Yang and

Ye (2008) — where the intensity of search is *lower* when average cost is greater than in the previous period.

■ **Equilibrium selection.** We make no claim of uniqueness of equilibria, even within a class of equilibria: for example, there exists a continuum of separating equilibria (with different values of α). A simple Pareto criterion selects a unique separating equilibrium, the one with the small fraction of active buyers required for c_L sellers not to mimic c_H sellers. Still, we found pooling and separating equilibria, as well as hybrid combinations of pooling and separating equilibria. We don't see this necessarily as a worrisome outcome: different industries may feature different equilibria.

■ **Price dynamics.** Empirical evidence regarding price dynamics reveals a series of stylized facts.

- Peltzman (2000) shows that, in a significant number of industries, prices rise faster than they fall. This is consistent with the first of our semi-separating equilibria.
- Blinder et al. (1998), among others, shows that prices tend to be rather sticky, that is, invariant to cost changes and other changes. This is consistent with our pooling equilibrium, although admittedly it is also consistent with many other models, namely menu-cost models.
- Klenow and Kryvstov (2008) for the US and Dhyne et al (2004) for the Euro area show that price decreases are less frequent than price increases; and that the absolute value of price increases is smaller than the absolute value of price decreases. This empirical evidence seems consistent with the the second hybrid model we considered: pooling upwards, separating downwards.

In sum, one size does not fit all; and while evidence for different industries seems broadly consistent with different equilibria, we note that a relatively simple model as ours can produce fairly rich dynamics that are consistent with different observable facts.

■ **Price dispersion.** Most of the search literature is based on the idea of price dispersion: to the extent that different firms set different prices, a searcher expects to find a better price by searching than by not searching. By contrast, in all of our equilibria all sellers set the same list price p; and in some of these equilibria "search" (i.e., active buyers) does take place along the equilibrium path. How can there be rational search without price dispersion?

First, in our framework the incentive for becoming an active buyer is to obtain better prices from sellers, including the seller the buyer is currently attached to: even though there are no differences in the list prices set by the various firms, active buyers are offered prices q that are lower than list prices p. To the extent that $q < p$, there is price dispersion in equilibrium, but this is the difference in prices paid by active and passive buyers, not price dispersion in the traditional sense.

Second, in the various pooling equilibria we derived buyers are uncertain about the value of cost, and thus about the value of q they will be quoted if they become active. Therefore, from the buyer's perspective there is "dispersion" of q in the probabilistic sense of the word.

■ **Welfare.** In separating equilibria prices are fully informative; however, social welfare is lower than in a pooling equilibrium. This is due to the fact that, unlike the pooling equilibrium, a separating equilibrium requires active buyers along the equilibrium path. Our assumption of unit demands implies that there are no output distortions. This biases the welfare comparison in favor of the pooling equilibrium. More generally, the message is that there is a trade-off between informative prices and the costs of being an active buyer (which are required for prices to be informative). In this sense, our result is related to Benabou and Gertner (1993). They show that higher uncertainty in the aggregate component of costs leads to less informative prices but may increase welfare; we show that the same effect may result from switching from a separating to a pooling equilibrium.

In our absorbing-state dynamic equilibria, seller value is equal to s both under pooling and separating equilibria. This implies that the social loss from buyer activity (in the separating equilibrium) is entirely born out by buyers in the form of higher prices and actual "search" costs paid along the equilibrium path.

6. Conclusion

Most of the economics literature on search and price dispersion is centered on final-consumer markets, markets where consumers are price takers. However, many transactions in the economy take place in a business-to-business context, a context where buyers have some power to determine prices.

In this paper, we propose a model to address this type of markets. Although parsimonious, our model is consistent with different types of equilibria (pooling, separating, semi-separating). And these equilibria offer an explanation for various stylized facts regarding price dynamics, including in particular the property that consumers seek a better deal (become "active" buyers) when they observe a price increase.

A. Purification of mixed strategies

In this section, we argue that the mixed-strategies equilibrium in Proposition 1 may be thought of as the reduced form of an incomplete information game with equilibrium in pure strategies. In the tradition of Harsanyi (1973), suppose that each buyer's switching cost is given by $s+\zeta$, where ζ is uniformly distributed in $[-\epsilon,\epsilon]$. Let $\Gamma(\epsilon)$ be the game that is obtained from our initial game by adding this disturbance to search costs. In fact, the original game corresponds to $\Gamma(0)$.

An equilibrium of this incomplete information game can be obtained from the equilibrium in Proposition 1 as follows. When $c=c_L$, sellers set $p=p_L-\epsilon$; when c switches from c_L to c_H, $p=p_H-(2-\alpha)\,\epsilon$; and in subsequent periods, $p=p_H-\epsilon$. In this equilibrium, buyers make search decisions based on strict inequalities, and search takes place with the probabilities indicated by Proposition 1. In particular, when c switches from c_L to c_H, buyers with search cost $s+\zeta'$ are indifferent between being active and being passive, where ζ' is given by

$$p_H-\big(s+\zeta'\big)\,[1]=p_H-(2-\alpha)\,\epsilon$$

It follows that the fraction of buyers who search is given by

$$\mathbb{P}\big(\zeta<\zeta'\big)\,[1]=\frac{\zeta'+\epsilon}{2\,\epsilon}=\alpha$$

where the last equality follows from the buyer's indifference condition.

B. Results for decreasing-cost case

In this section we include the results corresponding to Propositions 1 and 2 for the case when the low-cost state is absorbing (rather than the high-cost state).

Proposition 5. *There exist bounds $\overline{\alpha}$ and $\underline{\alpha}$, such that, if $\alpha\in[\underline{\alpha},\overline{\alpha}]$, then a separating equilibrium exists. Passive (resp. active) buyers pay p_k (resp. q_k) when $c=c_k$ ($k=L,H$), where the values of p_k and q_k are given by (??) and (??). Passive buyers' beliefs that $c=c_H$ are given by $\beta=1$ if $p=p_H$ and $\beta=0$ otherwise. Buyers become active with probability α when the $p=p_H$.*

Proposition 6. *If $s\geq(1-\delta)\,(c_H-c_L)/(1-\delta+\delta\,\gamma_H)$, then there exists a pooling equilibrium: passive buyers pay*

$$p(t)=c_L+s+(1-\gamma_H)^t\left(c_H-c_L-\delta\,\frac{1-\delta+\delta\,\gamma_H}{1-\delta}\,s\right)$$

whereas active buyers in period t pay

$$q_H(t)=c_L+\Big(1-\delta\,(1-\gamma_H)^{t+1}\Big)\left(\frac{c_H-c_L}{1-\delta+\delta\,\gamma_H}+\frac{\delta\,s}{1-\delta}\right)$$

$$q_L=c_L$$

Along the equilibrium path, passive buyers believe that $c=c_H$ with probability

$$\beta(t)=(1-\gamma_H)^t$$

If a price different from $p(t)$ is observed at time t, then $\beta(t)=0$. Buyers are passive along the equilibrium path.

C. Proofs

Proof of Proposition 1: First consider a c_L type trying to mimic a c_H type. This will change the value function only in the periods where the cost is low. The incentive compatibility (IC) condition is given by

$$(1-\alpha)\left(\frac{p_H - c_L}{1-\delta(1-\gamma_L)}\right) < \left(\frac{p_L - c_L}{1-\delta(1-\gamma_L)}\right)$$

The left-hand side (LHS) is the discounted expected payoff until cost changes to $c = c_H$ given that the first sets a $p = p_H$. The right-hand (RHS) side is the corresponding discounted payoff given that the firm sets a low price. The LHS decreases in α while the RHS is constant. Equating the two we find a lower bound on α for this IC condition to be satisfied:

$$\underline{\alpha} = \frac{c_H - c_L}{c_H - c_L + (1-\delta(1-\gamma_L))\, s}$$

Now consider the case of a c_H type deviating to mimic a c_L type. The one-step deviation rule applies, and so the IC condition simply becomes

$$p_L - c_H < (1-\alpha)\,(p_H - c_H)$$

This gives us an upper bound on α

$$\overline{\alpha} = \frac{c_H - c_L}{(1-\delta(1-\gamma_L))\, s}$$

Note that $\underline{\alpha} \in (0,1)$ and $\underline{\alpha} < \overline{\alpha}$. It follows that $\left[\max(0,\underline{\alpha}), \min(1,\overline{\alpha})\right]$ is a well defined interval. ■

Proof of Proposition 2: For buyers who have been passive in previous periods, the no-deviation constraint holds by construction: in each period, they are indifferent between being active and passive. Next we consider the seller's incentives. First, we need to confirm that the seller's expected future profit is positive for all t: if that is not the case, then the seller is better off by charging an infinite price (thus making zero profits). Clearly the worst possible scenario for the seller is when the cost switches to c_H at $t = 1$. Discounted profit is then given by

$$\sum_{t=1}^{\infty} \delta^{t-1}\big(p(t) - c_H\big)$$

If this expected payoff is positive, then all other state realizations lead to a positive equilibrium payoff as well. This imposes a lower bound on the search costs:

$$s \geq (1-\gamma_L)(c_H - c_L)$$

To exclude any other deviations, we define buyer off-equilibrium beliefs to be the worst possible from the seller's point of view. That is, once a buyer observes a price that is off the equilibrium path, it updates its beliefs to $\beta = 0$ (that is, $c = c_L$ with probability 1). Consistent with the new beliefs, if the new price is above $p(0)$ — that is, above the buyer's indifference point — then all buyers become active. It follows that the seller loses all of its buyers, so such deviation is not profitable. Deviating to a lower price is equally not profitable: although it does not result in a loss of buyers, it lowers the profit margin on all transactions. ■

Proof of Proposition 3: Combining the structures of the two previous separating equilibria (for $\gamma_H = 0$ and $\gamma_L = 0$), the present equilibrium requires that buyers become active with probability α_L whenever price increases from p_L to p_H; and become active with probability α_H whenever c remains at c_H. As before, equilibrium prices solve the system of equations

$$\begin{aligned} p_L &= q_L + s \\ p_H &= q_H + s \\ 0 &= q_L - c_L + \delta\left((1-\gamma_L)\, v_L + \gamma_L (1-\alpha_L)\, v_H\right) \\ 0 &= q_H - c_H + \delta\left((1-\gamma_H)\,(1-\alpha_H)\, v_H + \gamma_H\, v_L\right) \\ v_L &= p_L - c_L + \delta\left((1-\gamma_L)\, v_L + \gamma_L (1-\alpha_L)\, v_H\right) \\ v_H &= p_H - c_H + \delta\left((1-\gamma_H)\,(1-\alpha_H)\, v_H + \gamma_H\, v_L\right) \end{aligned}$$

The first two equations correspond to the buyer's indifference conditions; the second pair of equations corresponds to the seller's zero-profit conditions; and finally, the last two equations correspond to the sellers' value functions at each state. Solving the system we obtain the equilibrium price levels in the proposition. We also find that $v_H = v_L = s$ (as in the increasing-cost and decreasing-cost cases). ■

Proof of Proposition 4: Let $\overline{c}_i$ be discounted expected value of future costs given that $c = c_i$, where $i = L, H$; that is,

$$\overline{c}_i = \mathbb{E}\left(\sum_{t=0}^{\infty} \delta^t\, c_t \;\middle|\; c_0 = c_i\right)$$

These values are recursively defined by

$$\overline{c}_i = c_i + \delta\Big(\gamma_i\, \overline{c}_j + (1-\gamma_i)\, \overline{c}_i\Big)$$

for $i, j = L, H$, $i \neq j$. Also, let

$$\overline{c} = \frac{\gamma_L\, c_H + \gamma_H\, c_L}{\gamma_L + \gamma_H} \tag{29}$$

be the unconditional steady state average cost.

In a pooling equilibrium buyers do not search along the equilibrium path. If search takes place and a buyer finds that $c = c_H$ (that is, $q = q_H$), then the buyer is more pessimistic than non-searchers regarding the value of c; and as a result no search takes place in the future. Since sellers Bertrand compete for a searcher, it follows that, if $c = c_H$, then the buyer expects to pay a stream of prices equal to the discounted value of future costs, that is,

$$u_H = \overline{c}_H$$

The price offered to searchers satisfies the seller's zero-profit condition

$$q_H(t) - c_H + \sum_{\tau=t+1}^{\infty} p(\tau) - \mathbb{E}\big(c(\tau)\big) = 0$$

or simply

$$q_H(t) = \overline{c}_H - \sum_{\tau=t+1}^{\infty} p(\tau)$$

If $c = c_L$ is observed upon search, then the buyer becomes more optimistic than other buyers regarding cost, and as a result searches in all subsequent periods until $c = c_H$. Buyer's value is then recursively given by

$$u_L = -c_L + \delta \big((1-\gamma_L)\, u_L + \gamma_L\, u_H - s \big)$$

Moreover, as before the price offered to searchers is given by $q_L = c_L$ (sellers expect to make no profits from a searcher who is expected to search again in the next period).

As for non-searchers, their beliefs $\beta(t)$ that $c = c_H$ are defined recursively as follows:

$$\beta(t) = (1-\gamma_H)\, \beta(t-1) + \gamma_L \big(1 - \beta(t-1)\big) \tag{30}$$

Non-searchers are indifferent between searching and not searching. This implies equilibrium prices

$$p(t) = (s - u_L)\,(1-\delta) + \big(\beta(t) - \delta\, \beta(t+1)\big)(u_L - u_H)$$

■

Proof of Proposition 5: As in the increasing-cost case, equilibrium prices solve the following system of equations:

$$p_L = q_L + s \tag{31}$$

$$p_H = q_H + s \tag{32}$$

$$0 = q_L - c_L + \frac{p_L - c_L}{1-\delta} \tag{33}$$

$$0 = q_H - c_H + \frac{\delta\,(1-\alpha)\,(1-\gamma_H)(p_H - c_H)}{1 - \delta\,(1-\alpha)\,(1-\gamma_H)} + \frac{\delta\,\gamma_H\,(p_L - c_L)/(1-\delta)}{1 - \delta\,(1-\alpha)\,(1-\gamma_H)} \tag{34}$$

Equations (31) and (32) result from the buyers' indifference condition (to be active or passive); Equations (33) and (34) result from the sellers' zero profit condition (Bertrand competition for an active buyer).

Solving the system we obtain expressions for equilibrium prices.

$$p_L = c_L + (1-\delta)s \tag{35}$$

$$q_L = c_L - \delta s \tag{36}$$

$$p_H = c_H + (1-\delta)\, s + \alpha\,\delta\, s - \alpha\,\delta\,\gamma_H\, s \tag{37}$$

$$q_H = c_H - s\,\delta + \alpha\,\delta\, s - \alpha\,\delta\,\gamma_H\, s \tag{38}$$

Note that the buyers' equilibrium strategy is optimal by construction (that is, prices were derived so that buyers are indifferent between being active and passive). However, the sellers' optimal strategy depends on α, the fraction of active buyers when price is high. From (35)–(36) and (37)–(38) we can construct the incentive compatibility conditions:

$$(1-\alpha)\left(\frac{p_H - c_H + \delta\,\gamma_H\, \frac{p_L - c_L}{1-\delta}}{1 - \delta\,(1-\gamma_H)\,(1-\alpha)}\right) \geq \frac{p_L - c_L}{1-\delta} - \frac{c_H - c_L}{1 - \delta\,(1-\gamma_H)} \tag{39}$$

$$\frac{p_L - c_L}{1-\delta} \geq (1-\alpha)\left(p_H - c_L + \delta\, \frac{p_L - c_L}{1-\delta}\right) \tag{40}$$

Let $\overline{\alpha}$ and $\underline{\alpha}$ be the solutions with respect to α of (39) and (40) as equalities (respectively). Computation establishes that $0 < \underline{\alpha} < 1$ and $\underline{\alpha} < \overline{\alpha}$, which implies that the interval $\left[\underline{\alpha}, \min\left(1, \overline{\alpha}\right)\right]$ is non-empty. ■

Proof of Proposition 6: In equilibrium, price increases to the highest level p_H whenever costs changes from c_L to c_H. Let t be the number of periods elapsed since the last time price increased. Buyers' beliefs regarding the cost level are then given by

$$\beta(t) = (1 - \gamma_H)^t$$

As before, buyers are indifferent between being active and being passive. The expected value from being an active buyer depends on whether cost is high or low:

$$u_H = -\frac{c_L}{1-\delta} + \frac{c_L - c_H}{1 - \delta(1-\gamma_H)} \tag{41}$$

$$u_L = -\frac{c_L}{1-\delta} - \frac{\delta\, s}{1-\delta} \tag{42}$$

These value functions reflect an important difference with respect to the increasing-cost pooling equilibrium: an active buyer who finds that $c = c_L$ is more optimistic about costs than a passive buyer. Since the latter is indifferent between being active or passive, it follows that active buyers who find that $c = c_L$ are strictly better off by continuing to be active in all future periods until cost switches to c_H. (By contrast, in the increasing-cost case, a searcher who finds that cost has switched to its absorbing state strictly prefers not to search in the future.)

The expected value from being a passive buyer is given by

$$u(t) = -p(t) + \delta \max\left\{u(t+1),\, \left(1 - \beta(t+1)\right)\, u_L + \beta(t+1)\, u_H - s\right\} \tag{43}$$

As before, sellers set a price such that buyers are indifferent between being active and passive. This implies that $u(t)$, the value for a buyer who does not search, is equal to the expected value from searching:

$$u(t) = \left(1 - \beta(t)\right) u_L + \beta(t)\, u_H - s \tag{44}$$

Together, (41)–(44) determine $p(t)$.

By construction, passive buyers have no profitable deviation. Regarding sellers, one possible deviation is to set $p = \infty$ and make no sales. We thus must check that sellers make positive discounted expected profits in all states. Consider the seller's worst-case scenario, namely the case when cost has been high for a "very long" time. In the limit, price converges to

$$\underline{p} \equiv \lim_{t\to\infty} p(t) = c_L + s$$

This leads to the participation constraint

$$\sum_{t=0}^{\infty} \delta^t \left(\underline{p} - (1-\gamma_H)^t\, c_H - \left(1 - (1-\gamma_H)^t\right)\, c_L\right) > 0$$

which in turn implies

$$s \geq \frac{1-\delta}{1-\delta+\delta\,\gamma_H}\,(c_H - c_L)$$

To exclude any other deviation from the equilibrium path we assume that buyers who observe an off-equilibrium price infer that cost is low. These beliefs are more optimistic (lower than $\beta(t)$) than the equilibrium beliefs. If in equilibrium buyers are indifferent between being active and being passive, then a price deviation induces buyers to search. This in turn leads to a subgame where profit per buyer is zero, and thus total profits are zero as well. Since in equilibrium sellers make positive profits at every subgame, it follows that any seller deviation is not profitable.

To complete the equilibrium, we derive the price offered to active buyers. As before, this price depends on the cost state (c_L or c_H); and is determined by the seller's zero discounted profit condition. ■

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