Incentive Pay and Systemic Risk

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**Abstract.** We show that, in the presence of correlated investment opportunities across firms, risk sharing between firm shareholders and firm managers leads to compensation contracts that include relative performance evaluation. These contracts bias investment choices towards correlated investment opportunities, thus creating systemic risk. Furthermore, we show that leverage amplifies all such effects. In the context of the banking industry, we analyze recent policy recommendations regarding firm managerial pay and show how shareholders optimally undo the policies’ intended effects.

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1. Introduction

There is a recent regulatory push in the U.S. from the SEC, the NYSE, and the U.S. government accompanied by the actions of consultants such as Institutional Shareholder Services to create, by means of relative performance evaluation (RPE), a tighter link between CEO pay and the factors under CEO control.\(^1\) This paper addresses the consequences of RPE for firm investment decisions and systemic risk in an industry model extension of Holmstrom (1982).

We propose a novel channel through which CEO incentive pay may have an effect on systemic risk. We consider the implications of relative performance evaluation, a practice that emerges in the equilibrium of our industry model. We show that RPE allows for a better alignment of interests between shareholders and managers, thereby reducing agency costs and rendering firms more productive; but it also leads managers disproportionately to choose investments that are correlated across firms, thus increasing systemic risk.

The analysis uncovers a series of strategic complementarities with important equilibrium implications. First, the more a CEO invests in correlated assets, the more rival CEOs want to do the same. Second, the more CEOs choose correlated assets, the more their own shareholders want to implement RPE contracts. Third, the more CEOs increase leverage, the more their own shareholders want to implement RPE contracts. In this manner, we suggest a novel channel through which leverage induces systemic risk.

Our model features competition between two firms. Each firm is owned by a risk-neutral principal (the firm’s shareholders) and managed by a risk-averse agent (the firm’s CEO). The agent is required to spend costly unobservable effort that increases the firm’s returns. The manager must also choose how to allocate the firm’s assets. Our central assumption is that each firm has access to two investment opportunities, one with only idiosyncratic risk and another one with risk that is correlated across firms. So as to focus on risk we assume both projects have the same expected return.

As in the classical principal-agent setting with hidden action, in our model the agent is induced to deploy unobservable effort by linking her pay to the firm’s performance. However, because the agent is risk-averse, her contract can be improved upon by incorporating RPE: making compensation depend on relative rather than absolute performance leads (in equilibrium) to lower pay volatility, especially when the common investment opportunities are highly correlated across firms.

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\(^1\) In a 2015 press release (“SEC Proposes Rules to Require Companies to Disclose the Relationship Between Executive Pay and a Company’s Financial Performance”), the SEC states that “The proposed rules would require companies to disclose in a new table ... the company’s total shareholder return (TSR) as well as the TSR on an annual basis of the companies in a peer group.” Institutional Shareholder Services (ISS), a leading corporate governance consultancy, adopts pay-for-performance screens to identify “companies that demonstrate a significant level of misalignment between the CEO’s pay and company TSR, either on an absolute basis or relative to a group of peers similar in size and industry.” See also the NYSE’s Listing Company Manual, Section 303A.05; as well as Regulation S-K Item 201(e) (Title 17: Commodity and Securities Exchanges).
The model’s novelty stems from the strategic interaction between firms and the endogeneity of the industry return. Relative performance compensation leads managers to put more weight on investments that are common to the rival firm, as opposed to firm-specific investments subject to idiosyncratic risks. Moreover, the weights placed by each firm in the common project are strategic complements: the more one firm invests in common investments, the more the other firm wants to do the same. A greater weight placed by both firms on the common project implies greater correlation of the firms’ overall returns. This greater correlation reduces the level of firm CEO’s risk for a given level of pay. This in turn is good news for our (risk-neutral) firm shareholders: the same level of agent utility can be offered with lower expected pay. We show that the induced strategic choice by shareholders of CEO contracts features strategic complementarity in the degree of RPE: if one firm designs a compensation package with more RPE, the optimal response of the rival firm’s shareholders is to increase the level of RPE in the compensation of their own manager.

Overall, our two-stage game of shareholder choice of compensation scheme followed by manager choice of portfolio composition leads to an equilibrium where RPE is chosen by shareholders and a disproportionate choice of common assets is chosen by managers. This in turn results in a higher level of systemic risk, an increased likelihood of joint firm failure caused by industry participants putting most of their eggs in the same basket.\footnote{Although we offer a very specific rationale for RPE, several of our results are also consistent with other motivations for RPE. For example, shareholders may resort to RPE as a means to attract higher-ability managers; or RPE may be useful in learning about managerial ability in a model with career concerns. However, given that RPE is in place (and regardless of the reasons for its existence), risk averse bank managers will tend to choose common assets, thus creating systemic risk.}

We then extend the model to allow for firm leverage. We show that with leverage, the manager is incentivized to invest more in both risky projects, to the extent that these earn a return higher than the borrowing rate. Because some of the risk associated with the correlated project can be hedged via RPE, the manager is offered more RPE, and engages in relatively more investment in the correlated project, than in the model without leverage.

We develop several empirical predictions of the model. Here we highlight three of them, which we believe are new in the literature. First, in the cross section, firms that have more leverage engage in both more investment in correlated projects and use more RPE to reduce the CEO’s exposure to this risk. The model thus identifies a novel channel through which leverage induces systemic risk. Second, the use of relative performance evaluation should vary over time with the availability of correlated projects. For example, stronger correlations in stock returns in market downturns should be associated with greater incentives to correlate strategies for closet indexers in the mutual fund industry. While the econometrician may not know at every point in time the information available to the board with respect to such investment opportunities for a general industry, ex-post the information may be revealed in the balance sheet. Another way to test this prediction is to use changes in the tax code or
in regulations that create opportunities for new products or markets, for example the lowering of barriers to interstate commerce, or other significant industry transformations including product innovation, and test how RPE changes subsequently. Third, as an effect of RPE, executive pay volatility decreases as industry volatility increases, all else constant. This prediction is new in that it relates directly to incentive pay as a source of herding behavior and helps identify our mechanism from other sources of correlated actions.

In the banking industry, the regulatory push for greater use of RPE described above is being met with a contemporaneous and unprecedented effort by central banks around the world to regulate CEO pay as part of a reform effort that followed the subprime crises in the U.S. and other countries. In the last part of the paper, we analyze the impact on systemic risk of many of the new regulatory actions by central banks that constrain CEO pay. We show that these policies are costly to shareholders who then optimally adjust RPE as a way to minimize this cost, effectively undoing the intended consequences of the systemic-risk reducing policies. We view this ineffectiveness result as a reflection of the argument put forth in Posner (2009, p. 297) that “Efforts to place legal limits on compensation are bound to fail, or to be defeated by loopholes, or to cause distortions in the executive labour market and in corporate behavior.”

More than a “loophole,” we argue that existing dimensions of executive pay will adjust to an artificial regulation of one dimension in isolation; and that, as a result, the intended goal of reducing systemic risk may fail to materialize; rather, a negative effect (a “distortion”) may take place in “corporate behavior.” Our paper thus adds to the growing literature on the unintended effects of banking regulation.3

Our paper presents a result originally conjectured in the path-breaking work of Holmstrom (1982). We do this by allowing the agent to affect the variance of noise in performance and the correlation with the (endogenous) benchmark. Maug and Naik (2011) also show that fund managers compensated with relative performance contracts engage in correlated strategies. Their analysis differs from ours in two important ways. First, they take the form of the managerial contract as given when analyzing investment choices. We show that RPE is an equilibrium outcome and moreover that the choice of RPE is a strategic complement to the amount invested in the correlated project. Second, in their set up there is only one firm with an agency problem, which means that there are no strategic complementarities across firms in equilibrium. We show that these strategic interactions lead to higher levels of systemic risk.

Celentani and Loveira (2006) and Ozdenoren and Yuan (2014) associate endoge-

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3. Murphy (2009) and Ferrarini (2015) hypothesize unintended consequences of regulating executive pay on the quality of the workforce and the productivity of the industry. Kleynenova and Tuna (2015) provide evidence that an unintended consequence of the increased regulation in the U.K. is that compensation contracts have become more complex for U.K. banks relative to other firms in the U.K. In the same spirit, (French et al., 2010) suggests that governments should not regulate the level of executive pay in financial firms because markets are better at setting prices.
nous executive compensation with endogenous investment choices, but in these models taking correlated actions can only occur when the industry is expected to perform well. We assume projects have the same expected return so as to focus on risk driven by pay packages. For the same reason, we ignore agency considerations such as those treated in Buffa et al. (2014). Zwiebel (1995) and Bhattacharya et al. (2007) model relative performance in contracts only on the firm’s upside and find that this asymmetric RPE leads certain firms to take actions that separate them from others. In Gumbel (2005) correlated actions occur only for high levels of managerial risk aversion.

A popular mechanism associated with the banking industry that generates correlated actions is a bailout guarantee (Kane, 2010; Acharya and Yorulmazer, 2008; Farhi and Tirole, 2012; Acharya et al., 2015). Correlated actions in the banking industry arise also: out of a desire to manage the private information conveyed by banks’ investment decisions (Acharya and Yorulmazer, 2008); because banks diversify their idiosyncratic risks by swapping assets (Allen et al., 2012); because banks have an incentive to fail and survive together, as a result of the recessionary spillover caused by the failure of one bank on surviving banks (Acharya, 2009); due to a need to meet future capital regulatory constraints (Martinez-Miera and Suarez, 2014).

Our paper is also related to a literature that studies spillovers in governance through compensation packages and the labor market for executives. As in our paper, Acharya and Volpin (2010) and Dicks (2012) show that compensation choices of firms are strategic complements and thus the weakening governance in one firm that raises pay to its CEO induces other firms to also raise pay to their CEOs and to weaken governance. Cheng (2011) shows that RPE can cause correlated choices in governance across firms when managers have career concerns. Levit and Malenko (2016) show that directors’ willingness to serve on multiple boards creates correlated choices in governance.

The remainder of the paper is structured as follows. Section 2 introduces the model’s basic ingredients, whereas Sections 3 and 4 solve for the model’s equilibrium. Specifically, Section 3 derives the managers’ optimal choice of effort and investment portfolio, while Section 4 analyzes the shareholders’ optimal choice of compensation contracts. Section 5 extends the basic model to the case when leverage is an endogenous variable determined by managers. In Section 6 we develop several model predictions and in Section 7 we use our results to shed light on a series of policy measures, including various restrictions on bank CEO pay. Section 8 concludes the paper.

2. Model

Consider an industry with two firms, denoted \( i = 1, 2 \). Suppose that firm \( i \)'s CEO has a utility function \(- \exp(-w_i + d_i)\), where \( w_i \) is CEO compensation and \( d_i \) the CEO’s disutility from effort \( e_i \). By assuming an exponential utility function, we assume firm CEOs are risk averse, and without loss of generality set risk aversion to 1. By
contrast, we assume firm shareholders are risk neutral. We revisit this assumption toward the end of Section 4.

Compensation is a linear function of own and rival firm performance:

\[ w_i = k_i + a_i r_i - b_i r_j \]  

where \( j \neq i \) and we assume \( a_i, b_i > 0 \) are compensation coefficients to be determined by shareholders as part of the CEO contract. In particular, \( b_i \) corresponds to relative performance evaluation, the central issue of our analysis.

We assume the CEO’s disutility of effort is quadratic:

\[ d_i = \frac{1}{2} \gamma_i e_i^2 \]

The firm’s return, \( r_i \), is a combination of: effort, \( e_i \); return on an activity of a type that is available to the whole industry, \( c_i \); and return on an activity that is available to the firm alone, \( s_i \). Until Section 5 we exclude the possibility of leveraging. This implies that each firm’s assets are equal to its equity; and that the CEO’s portfolio choice is limited to determining the fraction \( x_i \) of assets invested in common assets, where \( x_i \in [0, 1] \). We thus have

\[ r_i = e_i + x_i c_i + (1 - x_i) s_i \]  

It should be transparent in this formulation that the asset portfolio component of the return acts as noise on the observability of effort. This modeling choice differentiates our model from Holmstrom’s (1982); it is also the feature that allows the agent to control the correlation of firm returns with respect to industry returns, and thus to endogenize the impact of relative performance evaluation.

Since our focus is on risk and correlation induced by joint portfolio choices, we assume that all underlying assets have the same expected value and variance. Specifically, we assume that \( c_i \) and \( s_i \) are normally distributed with mean \( \mu \) and variance \( \sigma^2 \); and with no further loss of generality we assume \( \sigma^2 = 1 \).

Our crucial assumption regarding the underlying assets is that, while \( s_1 \) and \( s_2 \) are independent, \( c_1 \) and \( c_2 \) are positively correlated. Specifically, we denote by \( \psi \) the covariance of \( c_1 \) and \( c_2 \) and assume that \( \psi \in (0, 1) \). We also assume that \( s_i \) is independent of \( c_i \) (as well as \( c_j \) and \( s_j \)).

Our principal-agent-and-competition game proceeds as follows. In a first stage, risk-neutral shareholders simultaneously determine their CEO’s compensation parameters: \( k_i, a_i \) and \( b_i \). We assume that \((k_i, a_i, b_i)\) is observed by firm \( i \)’s CEO but not by other firms. This assumption reflects the fact that compensation contracts are typically observed with considerable noise. Next, CEOs simultaneously choose effort \( e_i \) and portfolio structure \( x_i \). Finally, Nature generates the values of \( c_i \) and \( s_i \); and payoff is paid.

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4. The optimality of linear contracts with relative performance is discussed in Holmstrom and Milgrom (1987) and Dybvig et al. (2010). Mirrlees (1999) demonstrates in certain contexts the non-optimality of linear contracts. In relation to this issue, below we discuss the implications of adding stock options with varying features to the contract.
We derive the Nash equilibrium of this multi-stage game, providing conditions such that the equilibrium exists and is unique; and compare it to the benchmark where RPE is not present (that is, \( b_i = 0 \)).

3. Portfolio choice without leverage

Substituting (2) for \( r_i, r_j \) in (1), we get

\[
 w_i = k_i + a_i \left( e_i + x_i c_i + (1 - x_i) s_i \right) - b_i \left( e_j + x_j c_j + (1 - x_j) s_j \right) \tag{3}
\]

It follows that the first and second moments of CEO compensation are given by:

\[
 \mathbb{E}(w_i) = k_i + a_i e_i - b_i e_j + (a_i - b_i) \mu \tag{4}
\]

\[
 \mathbb{V}(w_i) = a_i^2 x_i^2 + b_i^2 x_j^2 - 2 a_i b_i x_i x_j \psi + a_i^2 (1 - x_i)^2 + b_i^2 (1 - x_j)^2 \tag{5}
\]

Since \( w_i \) is linear in \( r_i \) and \( r_j \); and since the latter are normally distributed; it follows that the CEO’s utility maximization problem is equivalent to

\[
 \max_{e_i, x_i} \mathbb{E}(w_i) - \frac{1}{2} \mathbb{V}(w_i) - \frac{1}{2} \gamma_i e_i^2 \tag{6}
\]

The first-order condition with respect to \( e_i \) is given by

\[
 a_i - \gamma_i e_i = 0
\]

and so

\[
 e_i^* = \frac{a_i}{\gamma_i} \tag{7}
\]

where the asterisk denotes optimal (or best-response) value. This is a standard principal-agent result: effort is increasing in performance evaluation and decreasing in the disutility of effort parameter. We next move to the CEO’s optimal portfolio choice. The first-order condition with respect to \( x_i \) is given by

\[
 -a_i \left( a_i x_i - \psi b_i x_j \right) + a_i^2 (1 - x_i) = 0 \tag{8}
\]

(Notice the second-order condition is satisfied if and only if \( a_i > 0 \).) It follows that

\[
 x_i^* = \frac{1}{2} + \frac{\psi b_i x_j}{2 a_i} \tag{9}
\]

If there is no RPE — that is, if \( b_i = 0 \) — then \( x_i^* = \frac{1}{2} \). This corresponds to the standard result of risk lowering by portfolio diversification. Since the assets \( c_i \) and \( s_i \) are identically and independently distributed, it is optimal to split the portfolio equally across the two (a 50–50 split is also the solution if there is only one firm). By contrast, setting \( b_i > 0 \) induces a demand for hedging: by increasing the value \( x_i \), firm \( i \)'s CEO decreases the variance of its compensation. An immediate implication of (9) is that
**Proposition 1.** $x_i^*$ is increasing in $x_j$.

The intuition is that, under RPE (that is, with $b_i > 0$) choosing the common asset $c_i$ is a form of “insurance” by firm $i$’s CEO. Specifically, under relative performance evaluation, a high value of $c$ is bad news for firm $i$’s CEO to the extent that firm $j$’s CEO has chosen that asset. In order to hedge against this adverse outcome, firm $i$’s CEO optimally chooses to place a greater weight on asset $c$ as well. In other words, Proposition 1 states that $x_i$ and $x_j$ are strategic complements: firm $i$’s CEO benefits from investing in $c$ because firm $j$’s CEO does so. In fact, this allows us to characterize the equilibrium of the portfolio-choice game as well as its comparative statics with respect to performance evaluation parameters:

**Proposition 2.** If $a_i \geq b_i > 0$, then the portfolio-choice game has a unique equilibrium. Moreover, the equilibrium levels $\hat{x}_i$ are strictly increasing in $b_i$.

In other words, CEOs choose the common asset to the extent that rival CEOs choose the common asset and compensation is based on relative performance.\(^5\)

We now turn to the analysis of overall industry equity returns, which are given by

\[
R \equiv \sum_{i=1,2} (r_i - w_i) = \sum_{i=1,2} (e_i + x_i c_i + (1 - x_i) s_i - w_i)
\]

We define systemic risk as the variance of overall industry returns, $\mathbb{V}(R)$. In fact, given our distributional assumptions, an increase in $\mathbb{V}(R)$ is associated to an increase in tail risk. The next result, which is a corollary of Proposition 2, characterizes $\mathbb{V}(R)$.

**Proposition 3.** An increase in $b_i$ leads to an increase in systemic risk.

In words, Proposition 3 encapsulates one of our main results: relative performance evaluation may lead to an increase in systemic risk. The irony of Proposition 3 is that the increase in overall risk results from the CEOs desire to reduce their individual risk. In fact an increase in relative performance pay decreases managerial pay risk while increasing industry systemic risk.

### 4. CEO compensation

We now take one step back and consider the optimal (and equilibrium) choices by shareholders. Firm $i$’s shareholders, who we assume are risk neutral, choose $k, a_i, b_i$ so

\(^5\) Our assumption that $\mu_s = \mu_c$ is done primarily for expository purposes: we want to highlight the interaction between payoff variances and covariances portfolio choices. That said, we should note that our results are not knife-edged: even if $\mu_s$ is greater than, but approximately equal to, $\mu_c$, then the text in Proposition 2 still applies. A more general wording of the result would then avoid the $\mu_s = \mu_c$ assumption and rather be prefaced by the sentence: “There exists a $\epsilon > 0$ such that, if $|\mu_s - \mu_c| < \epsilon$, then...”
as to maximize the expected value of \( r_i - w_i \). Specifically, the maximization problem is given by

\[
\max_{k_i, a_i, b_i} \mathbb{E}(r_i - w_i) \\
\text{s.t.} \mathbb{E}(w_i) - \frac{1}{2} V(w_i) - d_i(e_i) \geq u_i \\
e_i = e^*_i(a_i) \\
x_i = x^*_i(a_i, b_i; x_j)
\] (11)

Our first result in this section provides conditions such that RPE emerges in equilibrium. First, we note that, from (9), portfolio choices are only a function of the ratio

\[ p_i \equiv b_i / a_i \]

That is, \( p_i \) measures the intensity of relative performance evaluation at firm \( i \). Given this definition, the best-response mapping (9) may be re-written as

\[ x^*_i = \frac{1}{2} \left( 1 + \psi p_i x_j \right) \] (12)

Equation (12) confirms Proposition 3: an increase in relative performance by firm \( i \) (measured by \( p_i \)) leads to an increase in \( x_i \) and \( x_j \): Equation (12) shows that the partial effect is to increase \( x_i \); and supermodularity implies that both \( x_i \) and \( x_j \) increase in the resulting subgame equilibrium. As one would expect, if \( p_i = 0 \), then the CEO’s optimal portfolio choice is \( x = \frac{1}{2} \); a mean-variance-utility CEO’s optimal portfolio is to place equal weights on i.i.d. projects.

**Proposition 4.** In equilibrium \( a_i, b_i > 0 \) (and so \( p_i > 0 \))

Risk-neutral shareholders are indifferent with respect to their firm’s portfolio composition. However, the need to compensate risk-averse CEOs leads shareholders to “internalize” the CEO’s risk aversion. Specifically, an increase in \( b_i \) leads to a decrease in the variance of CEO pay, which in turn allows shareholders to lower base pay. In other words, the thrust of Proposition 4 is that shareholders are willing to go along with the CEO’s desire to reduce risk; and relative performance evaluation enables CEOs to follow a risk-reducing portfolio strategy.

This result is not an artifact of contract linearity. Suppose shareholders were to give stock options to the CEO. Such options give incentives to build volatility in the firm’s own stock returns, which is accomplished by concentrating investments in any one of the two projects (since they are independent and have the same volatility). With RPE, CEOs would have a preference for the common project as it increases the correlation of returns and reduces the volatility of pay (while having no effect on the volatility of the underlying stock options). As shareholders want to reduce volatility in pay, RPE is optimal. Following this reasoning, a contract with an option component and RPE may in fact be optimal because part of the risk resulting in pay through the option would be hedgeable with RPE, perhaps resulting compensation
that is less expensive for the shareholder. We do not pursue this here because of the complexity of the problem, but note that if anything this contract would make correlated investments even more extreme.

**Comparative statics.** Proposition 4 states that, in equilibrium, RPE is enacted. However, it does not say much regarding the level of RPE, \( p_i = a_i/b_i \), or regarding the equilibrium portfolios chosen by firm managers. The following result addresses these issues:

**Proposition 5.** There exists a unique symmetric equilibrium. It has the property that \( x \) and \( p \) are strictly increasing in \( \psi \), ranging from \((p = 0, x = \frac{1}{2})\) when \( \psi = 0 \) to \((p = 1, x = 1)\) when \( \psi = 1 \).

As expected, if \( \psi = 0 \), that is, if there is no correlation between the CEO’s outcome (even when they invest in the same asset), then there is no point in offering RPE \((p = 0)\): in fact, RPE would only add noise to the system without creating any additional incentive. In contrast, if \( \psi = 1 \) then all the risk in CEO pay resulting from the common project can be hedged with RPE, leading firms to invest exclusively in the common project.

**The strategic nature of relative performance evaluation.** Earlier we showed that CEO portfolio choices, \( x_i \), are strategic complements. A similar question may be asked regarding the shareholder choices of RPE, \( p_i \).

**Proposition 6.** There exist \( 0 < \psi' < \psi'' < 1 \) such that, if \( \psi < \psi' \) (resp. \( \psi > \psi'' \)), then \( p_1 \) and \( p_2 \) are strict strategic complements (resp. substitutes).

The simpler intuition for Proposition 6 corresponds to the case when \( \psi \) is small. When that is the case, an increase in \( p_2 \) leads to an increase in \( p_1 \): RPE choices are strategic complements. By (12), an increase in \( p_2 \) leads to an increase in \( x_2 \). Given that \( x_2 \) is greater, the potential for variance decrease by increasing \( x_1 \) is greater. As a result, the incentive for Firm 1’s shareholders to increase RPE also increase.

More formally, this result emerges along the following lines. As shown in the Proof of Proposition 4, the first-order condition for shareholder \( i \) payoff maximization with respect to \( b_i \) implies

\[
p_i = \frac{\psi x_i x_j}{x_j^2 + (1 - x_j)^2}
\]  

(13)

In other words, it’s as if shareholder \( i \) “anticipates” the values of \( x_i, x_j \) and, accordingly, adjusts the choice of \( p_i \). Now suppose that \( \psi \) is small, specifically close to zero. Then \( x_j \) is close to \( \frac{1}{2} \). It follows that a small change in \( x_j \) has little effect on the denominator of (13). Therefore, all of the action is in the numerator, which is increasing in \( x_i \) and \( x_j \). An increase in \( p_j \) leads to an increase in \( x_j \) (cf (12)), and supermodularity implies that \( x_i \) increases as well. Together, this implies an increase in \( p_i \).
At the opposite extreme, if $\psi$ is close to 1, then the denominator is increasing in $x_j$ (at a high rate), which more than compensates for the increase in the numerator and implies that the increase in $x_j$ leads to a decrease in $p_i$. The idea is that the increase in $x_j$ increases the variance in pay from choosing the common project to such a high level that shareholders are better off by placing less weight on relative payoff.

To put it differently, $x_j$ has two effects on the variance of firm $i$’s CEO pay: a variance effect (through $x_j^2$) and a covariance effect (linear in $x_j$). For low levels of $\psi$, $x_i$ and $x_j$ are close to $1/2$ and the covariance effect dominates: an increase in $p_j$ leads to an increase in $x_j$ and because $x_i$ and $x_j$ are strategic complements, leads to an increase in $x_i$; the covariance effect is stronger and shareholders of firm $i$ increase $p_i$. For high levels of $\psi$, $x_i$ and $x_j$ are close to one and the variance effect dominates: an increase in $p_j$ increases $x_j$ and $x_i$, but $p_i$ decreases so as to reduce risk exposure of the CEO of firm $i$ to the returns of firm $j$.

Our analysis so far has uncovered two important results. First, under RPE risk-averse managers bias their portfolio choices in the direction of correlated investments, thus increasing systemic risk. Second, risk-neutral shareholders find it optimal to design RPE contracts. These results are based on a series of assumptions which we maintain to simplify the analysis and focus on the essentials. That said, in the Online Appendix we consider a series of possible extensions. First, we allow shareholders to be risk averse, thus internalizing some of the cost associated with systemic risk. We show that the equilibrium level of RPE is smaller but still positive, so long as the degree of risk aversion is not too large. Second, we allow for the effect of managers’ effort to vary by type of project. In particular, it seems reasonable to expect effort to be more important in investment decisions with independent returns. We show that, if this is the case, then the bias in favor of correlated projects is smaller, though still positive, as long as the effect of managers’ effort is not too different across types of project. Finally, we consider the possibility of shareholders owning shares in both firms, a reality in the modern corporate world and a recent contentious research question. We show that joint ownership leads to an increase in RPE. Intuitively, an increase in RPE by firm $i$ leads to an increase in $x_i$ (investment in the common project). Everything else constant, this reduces the variance of firm $j$’s CEO, which in turn benefits firm $j$’s shareholders (who do not need to pay firm $j$’s CEO as much as when variance is higher). Common ownership internalizes this externality, leading to a higher level of RPE.

5. Leverage

Up to now we assumed that, in addition to effort, the firm manager’s choice is limited to the allocation of $1 across two different assets. This precludes the possibility of leverage. By contrast, in this section we assume that the firm’s assets, $x_{ci} + x_{si}$, may be greater than the firm’s equity, which we continue to assume is fixed at $1$.

Introducing leverage shows that some of the intuitions presented earlier are remarkably robust; it also brings new ideas to the fore. Accordingly, in this section we
focus primarily on differences with respect to the previous analysis.

Assuming that the firm is able to borrow at rate $r_b$, the firm’s equity return is now given by

$$r_i = e_i + x_{ci} \tilde{c}_i + x_{si} \tilde{s}_i + (1 - x_{ci} - x_{si}) r_b$$

We can then write

$$r_i = e_i + x_{ci} (\tilde{c}_i - r_b) + x_{si} (\tilde{s}_i - r_b) + r_b$$

or, defining $c_i = \tilde{c}_i - r_b$, $s_i = \tilde{s}_i - r_b$,

$$r_i = e_i + x_{ci} c_i + x_{si} s_i + r_b$$  \hspace{1cm} (14)$$

Asset allocations are constrained by $x_{ci}, x_{si} > 0$. Leverage occurs when $x_{ci} + x_{si} > 1$. Below we provide a necessary and sufficient condition for positive leverage. Our simple formulation of leverage (i.e., holding $r_b$ constant for different values of leverage), leads to two important features: (i) leverage increases mean equity returns because $\mathbb{E}(\tilde{c} - r_b) > 0$ and $\mathbb{E}(\tilde{s} - r_b) > 0$, and (ii) leverage increases the volatility of equity returns.

For simplicity, we maintain the assumptions that $\mu_i = \mu_c = \mu$, where $\mu$ is the expected value of $c_i$ and $s_i$; and that $\sigma_i = \sigma_c = \sigma = 1$. These assumptions allow us to focus on the strategic motives leading firm managers to choose a given portfolio (that is, motives different from each asset’s intrinsic value). Note that in the present context $\mu$ is somewhat different than before because it is an excess return over the risk free rate.Finally, we continue to assume that $\psi$ measures the correlation between the firms’ common project returns.

**Leverage ratios and balance sheet.** As mentioned earlier, our setup assumes that the firm has $1 of equity to invest. In the benchmark model (without leverage) the firm’s assets are given by $x + (1 - x) = 1$. With leverage, however, assets equal equity plus debt, and so total assets can be larger than equity. Specifically, assets equals $x_c + x_s$, whereas leverage equals $(x_c + x_s) - 1 > 0$ (a negative number means the firm holds cash or a safe asset).

In this more general framework, the fraction of assets invested in the common project is no longer a sufficient statistic of the CEO’s portfolio strategy (as in our benchmark model). Instead, we now express portfolio choices as percentages of total assets, $x_c + x_s$:

$$z \equiv x_c + x_s$$

$$x \equiv x_c / z$$

$$1 - x = x_s / z$$

The return

$$r_i = e_i + x_{ci} \tilde{c}_i + x_{si} \tilde{s}_i + (1 - x_{ci} - x_{si}) r_b$$
should therefore be interpreted as the return on equity, since

\[ e_i + x_{ci} \tilde{c}_i + x_{si} \tilde{s}_i \]

is now the return on assets,

\[ \frac{(x_c + x_s) - \$1}{\$1} = z - 1 \equiv l \]

is now the debt/equity ratio (as well as the degree of leverage), and \( r_b \) the return on debt.

**Compensation.** Similarly to (3), firm \( i \) manager’s compensation is given by

\[ w_i = k_i + a_i e_i + b_i r_j - b_i e_j \]

\[ = k_i + a_i e_i + b_i e_j + (a_i(x_{ci} + x_{si}) - b_i(x_{cj} + x_{sj})) \mu + (a_i - b_i) r_b \]

Similarly to (4)—(5), mean and variance of firm manager’s pay are given by

\[ \mathbb{E}(w_i) = k_i + a_i e_i - b_i e_j + (a_i(x_{ci} + x_{si}) - b_i(x_{cj} + x_{sj})) \mu + (a_i - b_i) r_b \]  
\[ \mathbb{V}(w_i) = a_i^2 x_{ci}^2 + b_i^2 x_{cj}^2 - 2 a_i b_i x_{ci} x_{cj} \psi + a_i^2 x_{si}^2 + b_i^2 x_{sj}^2 \]

**Leverage and portfolio composition.** Similarly to (6), the CEO’s utility maximization problem is now equivalent to

\[ \max_{e_i, x_{ci}, x_{si}} \mathbb{E}(w_i) - \frac{1}{2} \mathbb{V}(w_i) - \frac{1}{2} \gamma_i e_i^2 \]

Similarly to (7), the first-order condition with respect to \( e_i \) leads to

\[ \hat{e}_i = a_i / \gamma_i \]

Similarly to (9), the first-order condition with respect to \( x_{ci} \) implies

\[ x_{ci}^* = \frac{\mu + \psi b_i x_{cj}}{a_i} \]

The first-order condition with respect to \( x_{si} \), in turn, implies

\[ x_{si}^* = \frac{\mu}{a_i} \]

Notice that the strategic complementarity across firms is limited to investments in the common asset, \( x_{ci} \). This may suggest that portfolio composition is different in a world with leverage. However, our first result shows that, as a function of the degree of RPE, portfolio composition is the same with or without leverage.
Proposition 7. In a symmetric equilibrium and for given RPE ratios $p_i \equiv b_i/a_i$, the values of $x_i$ (portfolio composition) are invariant with respect to the degree of leverage.

Before, we forced the level of leverage to be zero, that is, we forced total assets to add up to $1$. The next result characterizes the endogenous value of leverage chosen by firm managers if they have the freedom to do so.

Proposition 8. In a symmetric equilibrium, firm leverage $\ell$ is given by

$$\ell = x_c + x_s - 1 = \frac{\mu}{a} \frac{2 - \psi p}{1 - \psi p} - 1$$

For a given $p$, $\ell$ is decreasing in $a$; conversely, for a given $a$, $\ell$ is increasing in $p$.

Intuitively, an increase in incentive pay, $a_i$, leads to a greater variance in CEO pay. The latter optimally adjusts to such an increase by lowering investment levels.\(^6\) Conversely, an increase in RPE, as measured by $p$, leads to lower variance in CEO pay, for the reasons described earlier. The CEO optimally adjusts to such a decrease in variance by increasing investments levels.

Solving the equation in Proposition 8, we conclude that $\ell > 0$ if and only if

$$\mu > a (1 - \psi p)/(2 - \psi p)$$

One interpretation of this inequality is that, if expected returns from investment are sufficiently high with respect to the compensation parameters $a$, $p$ and the correlation coefficient $\psi$, then the CEO optimally chooses a positive degree of leverage; or, alternatively stated, if $a$ is sufficiently low or $p$ is sufficiently high with respect to the value of $\mu$, then the CEO optimally chooses a positive degree of leverage. We note that the result that leverage exists in equilibrium is true also if there is only one idiosyncratic project, provided the mean return on that project is high enough.\(^7\)

We also note that RPE has an effect on systemic risk through two different channels. First, Proposition 3 states that an increase in $b$ leads to a portfolio composition that places greater weight on common projects; and Proposition 7 states that this

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\(^6\) An increase in $a_i$ also leads to a higher expected value of incentive pay, which in of itself would lead to higher investment levels; but the variance effect dominates.

\(^7\) Note that there are no costs of financial distress or limited liability if the firm cannot pay its borrowed capital. If there are costs of financial distress, then firm shareholders have an interest in committing ex-ante — that is, when the borrowed capital is raised — to a low level of risk, for those costs are internalized by them at that time. Hence, financial distress costs should lead to lower RPE, lower leverage, a smaller allocation bias to the common project and lower systemic risk.

Limited liability, whether for CEOs or shareholders, gives an incentive to increase the firm’s risk level, which is accomplished by concentrating investment in a single project. Under RPE, it is more advantageous to concentrate the investment in the common project, for that reduces the risk borne by the manager, thereby lowering the cost of managerial compensation and raising the level of effort deployed by the manager. In sum, limited liability is likely to strengthen the model’s results.
effect is invariant with respect to the degree of leverage. Second, Proposition 8 shows
that an increase in $b$ leads to an increase in the degree of leverage; and, for a given
composition of CEO portfolios, an increase in leverage amplifies the systemic risk
effect of CEO portfolio choices.

Proposition 8 implies that in the optimal contract high equity incentives serve to
limit risk-taking in order to limit the volatility of pay. It therefore acts as a constraint
on leverage. John and Qian (2003) find evidence of a negative association between
equity incentives and leverage in the banking industry.

We conclude with a result that corresponds to Proposition 5 in the model without
leverage. It does not provide a characterization as complete as that of Proposition
5, but shows that (a) RPE takes place in equilibrium; and (b) the degree of RPE is
increasing in the degree of correlation across common projects, $\psi$.

**Proposition 9.** There exists a $\psi' > 0$ such that, if $0 < \psi < \psi'$, then in a symmetric
equilibrium $p > 0$, $dp/d\psi > 0$, and $dx/d\psi > 0$. Moreover, the equilibrium value of
$p$ is higher than in a model with no leverage.

Similarly to Proposition 5, Proposition 9 shows that an increase in $\psi$ leads to an
increase in $x$ and in $p$. As in Proposition 5, an increase in $\psi$ broadens the opportunity
for reducing variance of CEO compensation by means of RPE. This leads risk-neutral
shareholders to offer RPE as an “inexpensive” means to pay CEOs; and in turn leads
CEOs to choose the common project as a means to reduce pay variance under RPE.

Proposition 9 adds the result that a switch from no leverage to leverage is associ-
ated with an increase in RPE. The intuition for this result is that leverage magnifies
the effects described in the preceding paragraph. Specifically, leverage increases the
stakes for CEOs in terms of compensation. This increases the scope for risk-neutral
shareholders to offer utility in the form of lower variance in compensation, which they
do by increasing the degree of RPE.

Unlike the results in Proposition 5, which are valid for all $\psi \in [0, 1]$, Proposition
9 is valid for low values of $\psi$ only. In the Online Appendix, we report on an extensive
numerical analysis of the model equilibrium for higher values of $\psi$. All in all, our
numerical analysis confirms the above effects.

Leverage increases systemic risk through two distinct channels: (i) a standard
mechanism whereupon holding portfolio composition constant, it allows assets to grow
above the equity value of $1$, thus magnifying systemic risk; (ii) a novel mechanism
arising from the fact that levered firms feature a higher level of RPE and thus invest
more in the correlated project.

One final thought regarding leverage is that the value of $x$ is monotonically increas-
ing in $\psi$ even when leverage is negative. Negative leverage, or equivalently hoarding
cash, could also help reduce risk in pay for the manager. However, this is inefficient as
it sacrifices the return from investing in the risky projects. In addition, RPE allows
shareholders to extract a higher level of effort from the CEO.
6. Empirical predictions

Our first prediction, which echoes Holmstrom (1982) and others, is that RPE should be prevalent in different industries. While the earlier literature on RPE produced mixed results across industries, more recent evidence from both the implicit and the explicit use of RPE suggests that the generality of firms use RPE in CEO pay (see, for example, Albuquerque (2009) on implicit RPE and Angelis and Grinstein (2015) on explicit RPE). The industries where RPE in CEO pay is found to be more prevalent are Utilities and Finance, in the latter case especially among large banks (Albuquerque, 2014; Angelis and Grinstein, 2015; Ilic et al., 2015). There is also evidence of herding behavior (also called closet indexing) in U.S. mutual funds and pension funds, with the tendency to herd mostly accounted by common momentum strategies and concentrated in more illiquid and opaque securities (Lakonishok et al., 1992; Grinblatt et al., 1995; Wermers (1999)). Cremers et al. (2016) document this behavior in international markets, and papers for developing markets find similar patterns with somewhat stronger herding effects (e.g., Kim and Wei (1999) for Korea and Raddatz and Schmukler (2013) for Chile).  

Second, our analysis predicts that firms that are more leveraged engage in RPE to a greater extent. All else equal, when firms lever up to take advantage of higher returns, if some of the available projects are correlated across firms, investing in such projects renders RPE more valuable. Because firms with more leverage invest more in correlated projects, the model also identifies a novel channel through which leverage induces systemic risk.

Third, we predict that the usage of RPE in executive pay should be accompanied by herding in the choice of risk exposure across firms, thus magnifying systematic risk. This prediction is consistent with the herding narrative in Bhattacharyya and Purnanandam (2011). They report that between 2000 and 2006 — that is, the period preceding the financial crisis — the idiosyncratic risk of US commercial banks dropped by half, while systematic risk doubled. Brunnermeier et al. (2012) identify some of the correlated actions that may have led to increased systemic risk. They document that banks increased their income from trading, investment banking and venture capital activities, all noncore, nontraditional income sources. This prediction is shared with the models of Acharya and Yorulmazer (2008) and Farhi and Tirole (2012) because there, too, an implicit bailout guarantee leads banks to take on correlated risk.

Fourth, we predict cross-industry variation in the joint occurrence of RPE and herding in investment choices, driven by the availability of correlated investment opportunities. With the exception of the money management industry where both the set of investment opportunities available to participant firms and their investment
choices are observable by third parties, it is challenging to the econometrician to know at every point in time and for other industries the projects available to firms as well as the capital allocated to each of them. An alternative is to focus on the changing correlation in investment opportunities that is likely to occur within industries using a conditional CAPM model with time-varying betas. Predictability in time variation in betas should covary positively with the use of RPE within an industry. Another way to test this prediction is to use changes in taxes or regulatory barriers that create opportunities for new products and markets, and test how RPE in firms from industries affected by such changes behaves subsequently. Examples of regulatory barriers include regulatory impediments to competition across different business lines or impediments to interstate commerce or international trade. Cuñat and Guadalupe (2009) show that the deregulation leading to increased competition in the US financial sector in the 1990s brought about more incentive alignment to CEO contracts (though not more total pay). More direct evidence on this prediction is given by the fact that the usage of RPE in banking has increased following the deregulation of banking in the early 1980s, accompanying a parallel increase in the pay-for-performance sensitivity of bank CEOs (Crawford, 1999).

Fifth, our model predicts that, as the result of the introduction of RPE, executive pay volatility decreases as industry volatility increases, all else equal. This prediction is new, to the extent that it related directly to executive pay as a source of industry systemic risk. It thus helps to identify our mechanism as separate from other sources of systematic risk, such as bailout guarantees in the particular case of the banking sector.

7. Public policy

This section builds on the model presented above to offer some perspective on a wave of regulations over banker’s pay by central banks around the world. This policy trend aims at restricting incentive pay packages in response to the view that they have led to excessive risk taking, ultimately contributing to the 2008 financial crisis (e.g., International Monetary Fund, 2014). Previewing our results, we show that these new regulations conflict with the push for greater RPE usage mentioned earlier in the Introduction to this paper: RPE allows for a more perfect alignment of interests between shareholders and managers, thereby reducing agency costs and rendering banks more productive; but it also leads managers disproportionately to choose investments that are correlated across banks, thus increasing systemic risk. As a caveat, the model presented above does not explicitly recognize the undesirability of systemic risk, rather we assume that its reduction is the regulator’s ulterior motive.

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9. There is a debate on the link between compensation and risk taking. Bebchuk et al. (2010), Bhagat and Bolton (2013), DeYoung et al. (2013), Kolasinski and Yang (2017), and Cai et al. (2010) have argued that the incentive component of pay may have caused excessive risk taking. In contrast, Cheng et al. (2015), Fahlenbrach and Stulz (2011), and Hagendorff et al. (2016) have disputed the link between firm risk and CEO compensation.
In 2009, the Financial Stability Board set a series of principles and standards regarding financial institutions’ compensation packages (FSB Principles for Sound Compensation Practice, 2009). These standards were formulated at a sufficient level of abstraction so as to allow agreement among member countries with different views. For example, with respect to pay structure, the FSB simply advocates the alignment of compensation with prudent risk taking, with the latter encompassing all types of risks.

In Europe the FSB standards were implemented through detailed rules enacted by primary legislation. The most important is the 4th Capital Requirements Directive (CRD IV, 2013), which states that variable compensation cannot exceed 100% of fixed pay, with at least 40% of it deferred for a minimum of three years, with exceptions indicated in Article 94, (g) (ii). The European Banking Authority (EBA) subsequently issued detailed technical standards to clarify and interpret the rules enshrined in CRD IV. The EBA takes a broad interpretation of variable compensation, including in it all compensation that is not contractually predetermined. It states that variable pay should be based on risk-adjusted performance and that the criteria to gauge performance may include measures of absolute performance as well as measures of relative performance vis-à-vis industry peers.

An extreme position is being taken by Israeli legislators, who have approved a cap on total pay of bank CEOs of 35 times the lowest salary paid by the firm, with a current value of cap at around 650,000 USD (Abudy et al., 2017).

In contrast to Europe and Israel, the US has followed a regulatory approach based on the ex-post supervision of banks to check for consistency of FSB principles on sound compensation policies. An exception is the salary cap of $500,000 imposed on financial institutions that accepted TARP financing. Some authors observe that most firms accepting TARP funding did so before February of 2009, when the final pay restrictions were announced (Cadman and Carter, 2012).

In this section we use the model developed in the previous sections to remark on the strengths and weaknesses of some of these public policy measures and proposals. Our analysis suggests that they grossly omit the role that RPE plays in creating systemic risk, as shown in the previous sections.

CEO compensation includes several components: specifically, total pay is equal to fixed pay, $k_i$, plus variable pay (or pay for performance), $a_i r_i - b_i r_j$. Variable pay, in turn, is equal to incentive pay, $a_i r_i$, plus RPE pay, $-b_i r_j$. In what follows, we consider regulations that address each of these components of CEO compensation.

### Caps on incentive pay

Consider first a cap in the form $a_i \leq \bar{a}$, that is, an upper bound on the own-performance variable pay coefficient. The following result provides an irrelevance result that speaks to the ineffectiveness of incentive pay regulation.

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10. The EBA also states that “relative measures could encourage excessive risk taking and need always to be supplemented by other metrics and controls” (Executive Summary, 44), but is unclear as to whether excessive risk refers to bank idiosyncratic risk or industry-wide risk.
Proposition 10. In the model without leverage, a cap on incentive pay does not change the level of systemic risk. In the model with leverage, there exists \( \psi' > 0 \), such that, if \( \psi < \psi' \), then imposing a binding cap \( a_i \leq \bar{a} \) results in an increase in leverage and in systemic risk.

Recall that in both models, the share of assets invested in the common project only depends on the ratio \( p_i \equiv b_i/a_i \); and \( p_i \) is thus a sufficient statistic for systemic risk. In the model without leverage, the variance of pay can be written as \( \mathbb{V}(w_i) = a_i^2 f(p_i, x^*_i(p_i), x_j) \), so the choice of \( b_i \), which minimizes the variance of pay, is proportional to the choice of \( a_i \). Thus, any active constraint on \( a_i \) leads to a proportional change in \( b_i \) that keeps \( p_i \) constant and systemic risk unchanged. In the model with leverage, an active constraint capping the value of \( a_i \) leads to a change in \( b_i \) that is less than proportional, and \( p_i \) increases. Intuitively, a lower \( a_i \) leads to an increase in leverage, for fixed \( p \) (see Proposition 8). The additional resources are used in both risky projects, but because of the benefits of RPE, \( p \) increases to induce the manager to allocate relatively more to the common project. Thus, a cap on \( a_i \) leads to an increase in systemic risk.

Strictly speaking the actual proposal in CRD IV is not to cap \( a_i \), but rather to cap variable pay at 100% of fixed pay, that is \( a_i r_i - b_i r_j \leq k_i \). This leads to a compensation level given by

\[
  w_i = k_i + \min \{ a_i r_i - b_i r_j, k_i \}
\]

The second component of pay is equivalent to the payout from shorting a put option with the put’s underlying being \( a_i r_i - b_i r_j \) and its strike price being \( k_i \). Under this constraint, compensation is weakly increasing and concave on \( a_i r_i - b_i r_j \). As the utility function is increasing and concave over \( w_i \), the utility function remains increasing and concave over \( a_i r_i - b_i r_j \). The shareholder therefore still cares about the negative effect that the volatility of \( a_i r_i - b_i r_j \) has on the manager’s utility, and will try to use RPE to reduce that volatility. While the specific implications from a constraint that introduces a kink in compensation are hard to derive analytically in our setting, the mechanism in the previous sections should still apply, generating investments in the common project that are strategic complements and that increase in the amount of RPE.

While the effect of incentive-pay regulation does not seem to improve systemic risk in the model, it may actually have a strictly negative overall efficiency effect. A binding constraint that causes \( a_i \) to be lower than the equilibrium outcome reduces...
effort by bank executives, thus lowering the value added of the financial industry. The specific point that caps in incentive pay can lead to lower effort in the banking industry has been made by several authors in different contexts (e.g., Bhagat et al., 2008; Murphy, 2009, 2013; Core and Guay, 2010). Other unintended consequences have also been discussed. For example, in Dittmann et al. (2011) average pay increases with caps on high powered incentives; in Hilscher et al. (2016) caps on ownership may lead to increased risk-taking; and in Asai (2016) caps on bonus pay may lead to more underinvestment.

We view our ineffectiveness result as an illustration of the argument put forth in Posner (2009, p. 297). The compensation package already offers significant flexibility for shareholders to adjust to an artificial regulation; and that, as a result, no positive effect will take place in terms of systemic risk; rather, a negative effect (a “distortion”) may take place in “corporate behavior.”

The above discussion comes with a caveat. Our relatively simple model of industry competition is purposely simple and ignores potentially important features of the banking industry. Some of these may provide an independent justification for caps on variable pay. That possibility notwithstanding, our results suggest a fundamental weakness of the proposed measures: since RPE can be used to reduce bank CEO compensation risk, it can also be used to undo at least partly the intended risk-reduction goal of a cap on incentive pay.

Finally, we note that in the model without leverage a cap on variable pay reduces mean total compensation. To see this, recall that the individual participation constraint is given by

$$\mathbb{E}(w_i) - \frac{1}{2} \mathbb{V}(w_i) - \frac{1}{2} \gamma_i \varepsilon_i^2 = u_i$$

In equilibrium

$$\mathbb{V}(w_i) = a_i^2 x_i^2 + b_i^2 x_j^2 - 2 a_i b_i x_i x_j \psi + a_i^2 (1 - x_i)^2 + b_i^2 (1 - x_j)^2$$

$$= a_i^2 \left( x_i^2 + p_i^2 x_j^2 - 2 p_i x_i x_j \psi + (1 - x_i)^2 + p_i^2 (1 - x_j)^2 \right)$$

Because the term in curved brackets remains unchanged with the cap on $a_i$ (recall that $p$ and $x$ are unchanged), $\mathbb{V}(w_i)$ decreases with the cap on incentive pay (that is, $\mathbb{V}(w_i)$ is increasing in $a_i$). Likewise $e$ also decreases. Hence, mean total compensation decreases. Intuitively, the executive in the model is risk averse and cares about volatility. If she faces lower volatility, she does not require as much total pay. This result (for the model without leverage) contrasts with some arguments that mean total pay will not decrease (e.g., Murphy, 2013). In the model with leverage, a cap on $a$ may result in an increase in leverage that increases volatility of total pay, in which case the executive requires greater compensation.

It is reasonable to think of imposing caps on the component of pay for peer performance, $b$, since that’s what’s causing the bias towards the common project and the increase in systemic risk. In fact, we can show that in both models (with and without leverage) an active cap on $b$ leads to a lower $p$. In the model without leverage, this translates into lower investment in the common project and lower systemic risk,
since the benefit of hedging is now lower for the executive. For the model with leverage, although the relative investment in the common project goes down with an active cap on $b$, it is not possible to sign the impact on the absolute level of investment, since a lower $b$ also leads to a lower $a$, which in turn pushes up leverage.

**Caps on total pay.** To analyze the implications of a cap on total pay, we re-solve the shareholders problem, (11), imposing an additional constraint on average pay. We get the following result:

**Proposition 11.** Consider a cap on total pay: $E(w_i) \leq v$, where $v > 0$. In the model without leverage, the equilibrium level of systemic risk remains unchanged. In the model with leverage, there exists a $\psi' > 0$ such that, if $\psi < \psi'$, then both leverage and systemic risk increase.

To understand the intuition for this result, recall that, at the shareholder’s optimum, bank managers are held to their outside option:

$$E(w_i) - \frac{1}{2} V(w_i) - d_i(e_i) = u_i$$

A regulatory cap on $E(w_i)$ must be compensated by a variation in $V(w_i)$ or $d_i(e_i)$. How do changes in incentive pay $a_i$ change these components of bank CEO utility? In the proof, we show that $dV(w_i)/da_i = 0$. The idea is that an increase in incentive pay is compensated by a decrease in leverage so as to maintain total variance constant. Given this, the only way to increase CEO utility is by reducing effort level, which can only be induced by a decrease in $a_i$. This reduction in incentive pay leads to an increase in leverage. Moreover, for a given level of $b_i$, it also leads to an increase in $p_i = b_i/a_i$, the index of RPE that determines systemic risk. (In the proof we show that changes in $b_i$ do not compensate for the change in $a_i$.)

In the Squam Lake Report (French et al., 2010), the authors recommend governments not to regulate the level of pay, partly due to the lack of evidence linking level of pay and risk-taking, and partly due to unintended consequences of regulating the level of pay, such as affecting the value added of the financial industry. Proposition 11 provides some support for this fear, to which we add the danger of further increasing leverage.

Strictly speaking the cap on total pay is not on ex-ante pay but on ex-post pay. The cap thus turns the pay of the executive into a short put option. Like the cap on incentive pay discussed above, the utility function remains increasing and concave over $a_i r_i - b_i r_j$, implying that the mechanism in the previous sections still applies and that the equilibrium should still deliver a bias towards the common project as well as RPE.

Caps on incentives or on total pay may work in the wrong way by increasing leverage. The next subsection discusses a more traditional “macro prudential” constraint that is more effective at curbing systemic risk in this model.
Caps on leverage. As mentioned in the paragraphs following Proposition 9, leverage contributes to systemic risk. It is therefore reasonable to think of regulations limiting leverage as a way to reduce systemic risk. Specifically, consider the policy of setting a cap on leverage, that is, $\ell \equiv x_{ci} + x_{si} - 1 < L$. What effect does this have on CEO choices and shareholder choices?

Proposition 12. Consider a cap on leverage: $\ell \leq L$. If the cap is binding, then a decrease in $L$ leads to (a) no change in the RPE ratio $p$; (b) no change in portfolio composition $x$; (c) an increase in variable pay (both $a$ and $b$); (d) a decrease in systemic risk.

A direct effect of a decrease in leverage is to decrease CEO risk. Given this, shareholders optimally react by increasing the risk level of CEO compensation through both higher $a$ and $b$, while keeping $p$ constant. As $a$ increases, CEO effort increases and so does productivity. A cap on leverage reduces systemic risk by reducing the traditional amplification effect of leverage on equity returns.

Summary. There is a growing literature that studies the effects of constraints on executive pay in various settings. Most of these papers are cast in the context of a single-bank model and thus fail to take into account strategic effects across banks in their design of compensation. Our focus on correlated actions as the driver of systemic risk points to the concern that regulators should have that in order to evaluate whether risk taking at the level of individual banks translates into systemic risk, one has to determine whether these risks — large and small — are diversifiable at the industry level. This motivates our choice of modeling the industry equilibrium.

In the Online Appendix we discuss additional policy implications related to deferred pay and options on indexed stock in CEO pay.

8. Conclusion

Our main point is that, under RPE pay, risk-averse CEOs are likely to invest in common projects as a means to reduce variance in pay. Anticipating such behavior, shareholders have an incentive to offer RPE as a means to reduce the expected value of CEO compensation required to satisfy the CEO’s participation constraint.

In other words, we uncover four sources of strategic complementarity: (a) under RPE pay, the more a CEO invests in a correlated project, the more the rival CEO wants to do the same; (b) the more shareholders offer RPE pay, the more the rival firm’s shareholders want to do the same; and (c) the more CEOs invest in correlated projects, the more shareholders want to increase the extent of RPE pay and vice-versa. Finally, (d) leverage adds another incentive to engage in RPE.

We derived a number of public policy implications of these results. One additional area that might be worth examining is international spillover effects. Suppose that two banks in two different countries (e.g., Spain and Belgium) compete in the same
market; and suppose that one of the countries (e.g., Belgium) enacts regulation that effectively reduces the level of investment in common assets. Even if the other country (Spain, in our example) does not impose a regulatory restriction on its banks, strategic complementarity leads the latter to decrease their investment in common assets, in tandem with Belgium banks.

We have assumed that project return and variance levels are exogenous. If the price of the correlated investment project goes up and its expected return goes down for the same level of variance as more money is put into it, acting like decreasing returns, then the incentive to take the correlated project is attenuated and so would our mechanism. This mechanism notwithstanding, it is possible that the increased investment of firms in some assets leads managers in those firms to increase real investment and generate a continued price increase as in the trading frenzy model of Goldstein et al. (2013). Alternatively, if by taking on similar strategies the variance of the correlated project increases (see for example Basak and Pavlova, 2013), then there would be an added incentive for more RPE and hence more investment in the correlated project, possibly leading to a positive feedback loop. These mechanisms deserve further attention and are left for future research. That said, we note that all of our results are based on strict inequalities and continuous functions. As such, the results are not knife-edged: they stand the addition of small general equilibrium effects as the above.
Appendix

Proof of Proposition 1: The proof follows by direct implication of (9).

Proof of Proposition 2: Proposition 1 implies that $x_i$ and $x_j$ are strategic complements. Moreover, from (9) and the assumptions that $b_i > 0$ and $a_i \geq b_i$

$$\frac{dx_i^*}{dx_j} = \frac{b_i x_j}{2 a_i} < \frac{b_i x_j}{a_i} \leq x_j \leq 1$$

It follows that the reaction curves have a slope of strictly less than 1, which implies there exists a unique equilibrium. Moreover, standard supermodularity results (e.g., Theorem 2.3 in Vives, 2000) imply that the equilibrium value $\hat{x}_i$ is strictly increasing in $b_i$.

Proof of Proposition 3: Shareholder $i$’s returns are given by $r_i - w_i$. It follows that industry returns are given by

$$R = \sum_i r_i - w_i$$

This may be re-written as

$$R = \sum_i r_i - (k_i + a_i r_i - b_i r_j) = \sum_i (1 - a_i + b_j) r_i - k_i$$

It follow that

$$\mathbb{V}(R) = \sum_i (1 - a_i + b_j)^2 \mathbb{V}(r_i) + (1 - a_i + b_i)(1 - a_j + b_i) \mathbb{C}(r_i, r_j)$$

where $\mathbb{V}$ and $\mathbb{C}$ denote variance and covariance, respectively. Moreover,

$$\mathbb{V}(r_i) = \mathbb{V}(e_i + x_i c_i + (1 - x_i) s_i) = x_i^2 + (1 - x_i)^2$$

$$\mathbb{C}(r_i, r_j) = \psi x_i x_j$$

Note that, from our previous analysis, $x_i > \frac{1}{2}$ and $\partial x_i / \partial b_i > 0$. This implies that $\partial \mathbb{V}(r_i) / \partial b_i > 0$, $\partial \mathbb{V}(r_i) / \partial b_j = 0$, and $\partial \mathbb{C}(r_i, r_j) / \partial b_i > 0$. This implies that an increase in $b_i$ leads to an increase in $\mathbb{V}(r_i)$ and $\mathbb{C}(r_i, r_j)$, which in turn leads to an increase in $\mathbb{V}(R)$. Moreover, the direct effect of an increase in $b_i$ is also to increase in $\mathbb{V}(R)$. The result follows.

Proof of Proposition 4: At the optimum, the first constraint in (11) holds as an equality (and determines the value of $k_i$). Moreover $E(r_i) = \mu + e_i$. The maximization problem is therefore equivalent to

$$\max_{a_i, b_i} \mu + e_i - \frac{1}{2} \gamma e_i^2 - \frac{1}{2} \mathbb{V}(w_i(x_i, x_j))$$

s.t.

$$e_i = \hat{e}_i(a_i, b_i)$$

$$x_i = x_i^*(x_j; a_i, b_i)$$

$$x_j = x_j^*(x_i; \hat{a}_i, \hat{b}_i)$$
or simply

\[
\max_{a_i, b_i} \mu + \hat{e}_i - \frac{1}{2} \gamma \hat{e}_i^2 - \frac{1}{2} \mathbb{V}(w_i(x_i^*, x_j^*))
\]  

(19)

where, for simplicity, we omit the arguments of \(\hat{e}_i, x_i^*\) and \(x_j^*\).

Consider the first-order condition with respect to \(b_i\). From (7), \(\hat{e}_i\) is not a function of \(b_i\) or \(x_j\). We thus focus on the partial derivative of \(\mathbb{V}(w_i)\) with respect to \(b_i\) as well as the effects through changes in \(x_i\).

From (4) we see that \(\partial \mathbb{E}(w_i) / \partial x_i = 0\). It follows that the first-order condition for (6) that corresponds to \(x_i^*\) is equivalent to \(d\mathbb{V}(w_i) / dx_i = 0\). Given our assumption that bank \(i\)'s compensation contract is not observed by bank \(j\)'s CEO, it follows that \(dx_j^* / db_i = 0\). In sum, the effects through CEO portfolio choices are zero. It follows that the first-order condition with respect to \(b_i\) is simply given by

\[
\frac{d\mathbb{V}(w_i)}{db_i} = \frac{\partial \mathbb{V}(w_i)}{\partial b_i} = 0
\]

From (5), this first-order condition is given by

\[
(a_i x_i \psi - b_i x_j) x_j - b_i (1 - x_j)^2 = 0
\]

which leads to

\[
b_i = \frac{\psi a_i x_i x_j}{x_j^2 + (1 - x_j)^2}
\]  

(20)

By the same argument as before, when computing the first-order condition with respect to \(a_i\) we can ignore the indirect effects through \(x_i\) and \(x_j\). We thus have

\[
(1 - \gamma_i e_i) \frac{de_i}{da_i} - \frac{1}{2} \frac{\partial \mathbb{V}(w_i)}{\partial a_i} = 0
\]  

(21)

From (7), \(e_i = a_i / \gamma_i\) and \(de_i / da_i = 1 / \gamma_i\). From (5)

\[
\frac{\partial \mathbb{V}(w_i)}{\partial a_i} = 2 x_i (a_i x_i - \psi b_i x_j) + 2 a_i (1 - x_i)^2
\]

Substituting the above equalities into (21) and simplifying, the first-order condition with respect to \(a_i\) is given by

\[
\frac{1 - a_i}{\gamma_i} - x_i (a_i x_i - b_i x_j \psi) - a_i (1 - x_i)^2 = 0
\]

Solving for \(a_i\), we get

\[
a_i = \frac{1 + \gamma_i \psi b_i x_i x_j}{1 + \gamma_i x_i^2 + \gamma_i (1 - x_i)^2}
\]  

(22)

Finally, (20) and (22), as well as the assumption that \(\psi > 0\), imply that \(a_i, b_i > 0\) for \(x_i, x_j > 0\). ■
**Proof of Proposition 5:** Symmetry implies that $x_i = x_j = x$ and $p_i = p_j = p$, which in turn implies that (9) turns into

$$x = \frac{1}{2} \left( 1 + \psi p x \right) \tag{23}$$

The first-order condition with respect to the relative-performance parameter $b_i$ is given by

$$\frac{\partial V(w_i)}{\partial b_i} = 2 b_i x_j^2 - 2 \psi a_i x_i x_j + 2 b_i \left( 1 - x_j \right)^2 = 0$$

In a symmetric equilibrium, $a_i = a_j = a$, $b_i = b_j = b$, $x_i = x_j = x$, $\gamma_i = \gamma_j$. Moreover, for ease of interpretation, we also substitute $p$ for $b/a$ (that is, $b = a p$). The $b$ first-order condition then becomes

$$-p x^2 + \psi x^2 - p \left( 1 - x \right)^2 = 0 \tag{24}$$

Moreover, from (23)

$$x = \frac{1}{2 - \psi p} \tag{25}$$

Substituting into (24) and simplifying we get

$$\psi^2 p^3 - 2 \psi p^2 + 2 p - \psi = 0 \tag{26}$$

Computation establishes that (26) has two imaginary roots and a real root. We thus have a unique equilibrium. Setting $\psi = 0$ (resp. $\psi = 1$) in (26) implies the real root $p = 0$ (resp. $p = 1$). Differentiating the left-hand side of (26) with respect to $p$ we get

$$3 \psi^2 p^2 - 2 \psi p + 2$$

This polynomial has two imaginary roots, thus always the same sign. We conclude that $p$ is strictly increasing in $\psi$, ranging from $p = 0$ to $p = 1$. ■

**Proof of Proposition 6:** The first-order condition with respect to $b_i$ implies:

$$(x_i \psi - p_j x_j) x_j - p_i \left( 1 - x_j \right)^2 = 0$$

Solving (12) for $x_i$, we get

$$\hat{x}_i = \frac{2 + \psi p_i}{4 - \psi^2 p_i p_j}$$

and

$$1 - \hat{x}_i = \frac{2 - \psi p_i (1 + \psi p_j)}{4 - \psi^2 p_i p_j}$$

Substituting into the first-order condition and simplifying,

$$\Phi_i \equiv \psi \left( 2 + \psi p_i \right) \left( 2 + p_j \right) - p_i \left( 2 + \psi p_j \right)^2 - p_i \left( 2 - \psi p_j \left( 1 + \psi p_i \right) \right)^2 = 0 \tag{27}$$
Differentiating with respect to \( p_i \), we get

\[
\frac{\partial \Phi_i}{\partial p_i} = \psi^2 (2 + \psi p_j) - (2 + \psi)^2 - (2 - \psi p_j (1 + \psi p_i))^2 \\
+ 2 p_i \left(2 - \psi p_j (1 + \psi p_i)\right) \psi^2 p_j
\]

At a symmetric equilibrium, \( p_i = p_j = p \). Moreover, Proposition 5 implies that \( p = 0 \) if \( \psi = 0 \) and \( p = 1 \) if \( \psi = 1 \). Therefore

\[
\frac{\partial \Phi_i}{\partial p_i} \bigg|_{\psi = 0} = -8, \quad \frac{\partial \Phi_i}{\partial p_i} \bigg|_{\psi = 1} = -6
\]

The implicit-function theorem implies that, in the neighborhoods of \( \psi = 0 \) and \( \psi = 1 \), the sign of the slope of \( B_i(p_j) \), shareholder \( i \)'s best-response mapping, is the same as the sign of \( \frac{\partial \Phi_i}{\partial p_j} \). Differentiating (27), we get

\[
\frac{\partial \Phi_i}{\partial p_j} = \psi^2 (2 + \psi p_i) - 2 \psi p_i (2 + \psi p_j) + 2 \psi p_i \left(2 - \psi p_i (1 + \psi p_j)\right) (1 + \psi p_i)
\]

which implies

\[
\frac{\partial \Phi_i}{\partial p_j} \bigg|_{\psi = 0} = 2 \psi^2, \quad \frac{\partial \Phi_i}{\partial p_j} \bigg|_{\psi = 1} = -3
\]

The result then follows by continuity. (Notice in particular that, at \( \psi = 0 \), \( \frac{\partial \Phi_i}{\partial p_j} = 0 \), but in the right neighborhood where \( \psi > 0 \) we have \( \frac{\partial \Phi_i}{\partial p_j} > 0 \).)

**Proof of Proposition 7:** From (17) and (18), we derive the value of \( x \), the relative weight of common assets in total assets:

\[
x_i^* = \frac{1}{2} + \frac{\psi b_i x_{cj}}{2 a_i z_i}
\]

It follows that, in a symmetric equilibrium, this is the same as (9).

**Proof of Proposition 8:** In a symmetric equilibrium, (17)–(18) imply

\[
x_c = \frac{\mu + \psi b x_c}{a} \\
x_s = \frac{\mu}{a}
\]

Adding up and simplifying, we get

\[
z = x_c + x_s = \frac{\mu}{a} \frac{2 - \psi p}{1 - \psi p}
\]

The result follows from taking partial derivatives.
Proof of Proposition 9: Bank $i$’s shareholders solve

$$\max_{k_i, a_i, b_i} \mathbb{E}(r_i - w_i)$$

s.t. \[ \mathbb{E}(w_i) - \frac{1}{2} \mathbb{V}(w_i) - \frac{1}{2} \gamma_i e_i^2 \geq u_i \]
\[ e_i = e_i^*(a_i) \]
\[ x_i = x_i^*(a_i, b_i; x_j) \]

Substituting the IR constraint, this becomes

$$\max \left\{ \mathbb{E}(r_i) - u_i - \frac{1}{2} \mathbb{V}(w_i) - \frac{1}{2} \gamma_i e_i^2 \right\}$$

s.t. \[ \mathbb{E}(r_i) = e_i^* + \mu x_{ci}^* + \mu x_{si}^* + r_b \]
\[ \mathbb{V}(w_i) = a_i^2 x_{ci}^2 + b_i^2 x_{cj}^2 - 2 \psi a_i b_i x_{ci} x_{cj} + a_i^2 x_{si}^2 + b_i^2 x_{sj}^2 \]
\[ a_i x_{ci}^* = \mu + \psi b_i x_{cj} \]
\[ x_{si}^* = \mu/a_i \]
\[ e_i^* = a_i/\gamma_i \]

The first-order condition with respect to $b_i$ is given by

$$-\frac{1}{2} \frac{d \mathbb{V}(w_i)}{db_i} + \frac{\partial \left( \mu x_{ci}^* - \frac{1}{2} \mathbb{V}(w_i) \right)}{\partial x_{ci}^*} \frac{\partial x_{ci}^*}{db_i} = 0$$

Note that

$$\frac{\partial x_{ci}^*}{\partial b_i} = \frac{\psi x_{cj}}{a_i}$$
$$\frac{\partial \mathbb{V}(w_i)}{\partial x_{ci}^*} = 2 \left( a_i x_{ci}^* - \psi b_i x_{cj} \right) a_i$$
$$\frac{\partial \mathbb{V}(w_i)}{\partial b_i} = -2 \left( \psi a_i x_{ci}^* - b_i x_{cj} \right) x_{cj} + 2 b_i x_{sj}^2$$

The first-order condition thus becomes:

$$-\frac{1}{2} \left( -2 \left( \psi a_i x_{ci}^* - b_i x_{cj} \right) x_{cj} + 2 b_i x_{sj}^2 \right) + \left( \mu - (a_i x_{ci}^* - \psi b_i x_{cj}) a_i \right) \frac{\psi x_{cj}}{a_i} = 0$$

Using the first-order condition with respect to $x_{ci}$, the equilibrium values of $x_{si}$ and of $x_{cj}$, and simplifying, we get

$$(\psi^2 - 1) b_i (x_{cj}^*)^2 + \frac{\psi \mu x_{cj}}{a_i} - b_i x_{sj}^2 = 0$$

or simply

$$(\psi^2 - 1) b_i \left( \frac{a_i + \psi b_j}{a_j a_i - \psi^2 b_i b_j} \right)^2 + \frac{\psi}{a_i} \frac{a_i + \psi b_j}{a_j a_i - \psi^2 b_i b_j} - \frac{b_i}{a_j^2} = 0$$
Imposing symmetry (that is, \( a_i = a_j, b_i = b_j \)),

\[
(\psi^2 - 1) b \left( \frac{1}{a - \psi b} \right)^2 + \frac{\psi}{a (a - \psi b)} - \frac{b}{a^2} = 0
\] (30)

The first-order condition with respect to \( a_i \) is given by

\[
(1 - \gamma_i e_i) \frac{d e_i}{d a_i} - \frac{1}{2} \frac{\partial V(w_i)}{\partial a_i} + \frac{\partial (\mu x_i^* - \frac{1}{2} V(w_i))}{\partial x_i^*} \frac{\partial x_i^*}{\partial a_i} = 0
\] (31)

Note that

\[
\frac{\partial x_i^*}{\partial a_i} = -a_i^{-2} (\mu + \psi b_i x_{cj})
\]
\[
\frac{\partial x_i^*}{\partial a_i} = -\mu a_i^{-2}
\]

Substituting these in (31); substituting the equilibrium values of \( x_{ci}, x_{si} \); and imposing symmetry (that is, \( \gamma_i = \gamma, x_{ci} = x_c, \) etc), (31) becomes

\[
\frac{1 - a}{\gamma} - \frac{\mu^2 (2a - \psi b)}{a^2 (a - \psi b)} = 0
\] (32)

Substituting \( p \) for \( b/a \) in (30) and (32), we get a system of equations defining the equilibrium values of \((a, p)\):

\[
A \equiv (1 - a) \frac{a^2}{\gamma} - \frac{\mu^2}{\gamma} \frac{2 - \psi p}{1 - \psi p} = 0
\] (33)

\[
B \equiv (\psi^2 - 1) p \left( \frac{1}{1 - \psi p} \right)^2 + \frac{\psi}{(1 - \psi p) a} - p = 0
\] (34)

If \( \psi = 0 \), then \( b = p = 0 \) and \( a = a_0 \). Given this, we can take partial derivatives of \( A \) and get

\[
\frac{\partial A}{\partial \psi} \bigg|_{p = 0 \atop \psi = 0} = \frac{\partial A}{\partial p} \bigg|_{p = 0 \atop \psi = 0} = 0
\]

It follows by the implicit function theorem that

\[
\frac{d a}{d \psi} \bigg|_{p = 0 \atop \psi = 0} = 0
\]

We can therefore apply the implicit function theorem to (34) and get

\[
\frac{d p}{d \psi} \bigg|_{p = 0 \atop \psi = 0} = -\frac{1}{a_0} \times -2 > 0
\]

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It follows that for $\psi$ greater than, but different from zero, $p$ is positive and increasing in $\psi$. By Propositions 5 and 7, the same is true of $x$.

Consider now the model without leverage. Totally differentiating (24) at $\psi = p = 0$, we get

$$dp = \frac{x^2}{x^2 + (1 - x)^2} d\psi$$

(Notice the derivative with respect to $x$ multiplies $\psi$.) Since $x = \frac{1}{2}$ when $\phi = 0$, we get

$$\left.\frac{dp}{d\psi}\right|_{p = 0, \psi = 0} = \frac{1}{2}$$

This is smaller than the corresponding derivative in the model with leverage if and only if $a_0 < 1$, where $a_0$ is the equilibrium value of $a$ when $\psi = 0$. Substituting 0 for $p$ and $\psi$ in (32), we get

$$(1 - a) a^2 = 2 \mu^2 \gamma$$

which implies that $0 < a_0 < 1$.

**Proof of Proposition 10:** Consider first the model without leverage. Assume that the constraint on incentive pay is active, $a_i = a$, otherwise there would be no change in the game’s equilibrium outcome. Note that the equilibrium is still characterized by the solution $(p, x)$ that solves (24), because (24) results from the first-order condition for $b_i$, which still holds with equality. Once the equilibrium value of $p$ is determined, $b_i$ (and $b_j$) can be appropriately adjusted for any given $a_i$ (and $a_j$). Thus, a binding constraint on $a$ affects the value of $b$ but not the value of $p$. It follows that portfolio choices $x$ remain unaltered, keeping the level of systemic risk unchanged.

Consider now the model with leverage. Given that the constraint on $a$ is binding, the equilibrium value of $b$ is determined by (34) where the value of $a$ is treated as an exogenous parameter (basically $a = \bar{a}$). Applying the implicit function theorem, we get

$$\left.\frac{dp}{da}\right|_{p = 0, \psi = 0} = -\frac{\partial B / \partial a}{\partial B / \partial p} \bigg|_{p = 0, \psi = 0} = -\frac{\psi}{a^2}$$

This is zero at $\psi = 0$, but approaches zero from negative numbers. Hence, by continuity $dp/da < 0$ for low enough $\psi$. With lower $a$ and higher $p$, $x_c$ increases and so does $x_s$. Leverage increases and so does systemic risk.

**Proof of Proposition 11:** Suppose that $\mathbb{E}(w_i^*) > v$, where $w_i^*$ corresponds to the unconstrained solution. Then the cap matters, that is, $\mathbb{E}(w_i^*) = v$. Consider first the model without leverage. Then (11) may be written as

$$\max_{a_i, b_i} e_i - v$$
subject to the participation constraint,
\[ v - \frac{1}{2} V(w_i) - d_i(e_i) \geq u_i \]
as well as the constraint that \( e_i \) and \( x_i \) belong to the best-response mappings.

Notice that \( b_i \) is not present in the objective function: from (7), \( e_i \) is a function of \( a_i \) but not \( b_i \). It follows that the optimal \( b_i \) maximizes the slack in the participation constraint. As shown in the proof of Proposition 4, this implies \( \partial V(w_i) / \partial b_i = 0 \), which in turn determines the value of \( p_i = b_i / a_i \). It follows that the same value of \( p_i \) obtains as in the problem without the cap on pay.

Consider now the model with leverage. The problem faced by shareholders is:
\[
\max_{k_i, a_i, b_i} \mathbb{E}(r_i - w_i)
\]
subject to
\[
\mathbb{E}(w_i) - \frac{1}{2} V(w_i) - d_i(e_i) \geq u_i
\]
and that \( e_i, x_c \) and \( x_s \) belong to the best-response mappings. Let \( k_i \) be such that \( \mathbb{E}(w_i) = v \) and rewrite the problem as:
\[
\max_{a_i, b_i} \mathbb{E}(r_i) - v
\]
subject to
\[
v - \frac{1}{2} V(w_i) - \frac{1}{2} \gamma_i^{-1} a_i^2 \geq u_i
\]
Let \( a_i = f(b_i; v, x_{cj}, x_{sj}) \) be the solution to (36), as an equality, with respect to \( a_i \) (note that bank \( i \)'s shareholders take \( x_{cj} \) and \( x_{sj} \) as given). Also, recall that
\[
\mathbb{E}(r_i) = e_i^* + \mu x_{cj}^* + \mu x_{si}^* + r_b
\]
Then using (7) we can re-write (35)–(36) as
\[
\max_{b_i} \left\{ \gamma_i^{-1} f(b_i) + \mu \left( \frac{\mu}{f(b_i)} + \psi \frac{b_i}{f(b_i)} x_{cj} \right) + \frac{\mu^2}{f(b_i)} + r_b - v \right\}
\]
In order to maximize (37) we must compute the derivative of \( f(b_i) \), where \( a_i = f(b_i; v, x_{cj}, x_{sj}) \). Since this is derived from (36), which includes \( V(w_i) \), we must compute the derivatives of \( V \) with respect to \( a_i, b_i \).

\( \Box \) Partial derivatives of \( V(w_i) \) with respect to \( a_i \) and \( b_i \). First we show that \( dV(w_i) / da_i = 0 \). Note that this derivative takes \( x_{cj} \) and \( x_{sj} \) as given because we’re working with the problem of bank \( i \)'s shareholders and assume that bank \( i \)'s contract
is not observed by bank j’s CEO. Taking the derivative of (16) with respect to $a_i$, we get

$$\frac{dV(w_i)}{da_i} = 2 a_i x_i^2 - 2 \psi b_i x_i x_j + 2 a_i x_i^2 - 2 a_i^2 x_i \frac{\mu + \psi b_i x_j}{a_i^2}$$

$$+ 2 \psi a_i b_i x_j \frac{\mu + \psi b_i x_j}{a_i^2} - 2 a_i^2 x_i \frac{\mu}{a_i^2}$$

$$= 2 a_i x_i^2 - 2 \psi b_i x_i x_j + 2 a_i x_i^2 - 2 a_i^2 x_i^2 + 2 \psi b_i x_j x_i - 2 \mu x_i$$

$$= 2 a_i x_i^2 - 2 \mu x_i$$

$$= 2 a_i \frac{\mu^2}{a_i^2} - 2 \mu x_i$$

$$= 0$$

where we substitute (18) for $x_i$. We next compute the value of $dV(w_i)/db_i$. Taking the derivative of (16) with respect to $b_i$, we get

$$\frac{dV(w_i)}{db_i} = 2 (1 - \psi^2) b_i x_j^2 + 2 b_i x_j^2$$

$$= 2 (1 - \psi^2) b \left( \frac{\mu}{a - \psi b} \right)^2 + 2 b \left( \frac{\mu}{a} \right)^2$$

where we substitute (17) and (18) for $x_i$ and $x_j$. □

We next use these derivatives, $\partial V(w_i)/\partial a_i$ and $\partial V(w_i)/\partial b_i$, evaluated at the equilibrium values, in the solution to (37). The first-order condition for is given by

$$\frac{dE(r_i)}{db_i} + \frac{dE(r_i)}{da_i} \frac{df}{db_i} = 0$$

or

$$\psi \frac{\mu}{a_i} x_i + \left( \gamma_i^{-1} - \mu \left( \frac{\mu}{a_i^2} + \psi \frac{b_i}{a_i^2} x_i \right) - \frac{\mu^2}{a_i^2} \right) \frac{df}{db} = 0$$

Applying the implicit function theorem to (36) as an equality, we get

$$\frac{df}{db} = -\frac{\frac{dV(w_i)}{db}}{\frac{dV(w_i)}{da} + \gamma_i^{-1} a_i}$$

Substituting for $df/db$ in to (38) and simplifying we get

$$\mu \psi x_i - \left( 1 - 2 \frac{\mu^2}{a_i^2} - \mu \psi \gamma_i \frac{b_i}{a_i^2} x_j \right) \frac{1}{2} \frac{dV(w_i)}{db} = 0$$

(39)

Dropping the bank indexes $i, j$; substituting the result for $dV(w)/db$ in (39); substituting $p$ for $b/a$; and simplifying, we get

$$\psi - \left( 1 - \gamma \frac{\mu^2}{a^2} \frac{2 - \psi p}{1 - \psi} \right) \left( \frac{1 - \psi^2}{1 - \psi p} + 1 - \psi p \right) p = 0$$

(40)
Ultimately, we want to derive an equilibrium expression including $p$ (endogenous variable) and $\psi$ (exogenous parameter). The above expression includes another endogenous variable, $a$. We have another equation from which the value of $a$ can be obtained: (36), written as an equality. This expression includes the term $V(w_i)$, which is given by (16). Imposing symmetry, this becomes

$$(a^2 + b^2) (x_c^2 + x_s^2) - 2 \psi a b x_c^2$$

Substituting (17)–(18) (with subscripts $i, j$ dropped) for $x_c$ and $x_s$, and simplifying, we get

$$V(w_i) = \mu^2 \left( \frac{1 + p^2 - 2 \psi p}{(1 - \psi p)^2} + 1 + p^2 \right) \equiv g(p)$$

Substituting for $V(w_i)$ in (36), written as an equality, and solving for $a^2$, we get

$$a^2 = 2 \gamma (v - u) - \gamma g(p)$$

Substituting (42) for $a^2$ in (40), we get

$$0 = \Phi(p,v) \equiv$$

$$= \psi - \left( 1 - \frac{\gamma \mu^2 (2 - \psi p)}{(2 \gamma (v - u) - \gamma g(p)) (1 - \psi p)} \right) \left( \frac{1 - \psi^2}{1 - \psi p} + 1 - \psi p \right) p$$

We next compute this derivative at $\psi = 0$. Recall that $\psi = 0$ implies $b = p = 0$.

$$\frac{\partial \Phi}{\partial p} \bigg|_{p = 0, \psi = 0} = -2 \left( 1 - \frac{\mu^2}{v - u - \mu^2} \right) < 0$$

It follows from the implicit-function theorem that the sign of $dp/dv$ is the same as the sign of $\partial \Phi/\partial v$. From (43), we get

$$\frac{\partial \Phi}{\partial v} \bigg|_{p = 0, \psi = 0} = -2 \left( \frac{\mu}{v - u - \mu^2} \right)^2 p$$

Although this expression equals zero when $\psi = p = 0$, it converges to zero by means of a sequence of negative values as $\psi \to 0^+$. Therefore, there exists a $\psi' > 0$ such that, if $\psi < \psi'$, then $dp/dv < 0$.

Next we consider the effects of $v$ on leverage $z$. Taking total derivatives,

$$\frac{dz}{dv} \bigg|_{p = 0, \psi = 0} = \frac{\partial z}{\partial a} \bigg|_{p = 0, \psi = 0} \frac{\partial a}{\partial v} \bigg|_{p = 0, \psi = 0} + \frac{\partial z}{\partial p} \bigg|_{p = 0, \psi = 0} \frac{\partial p}{\partial v} \bigg|_{p = 0, \psi = 0}$$

From (28),

$$\frac{\partial z}{\partial a} \bigg|_{p = 0, \psi = 0} = -\frac{2 \mu}{a^2}$$

$$\frac{\partial z}{\partial p} \bigg|_{p = 0, \psi = 0} = 0$$
From (42), \( \partial a / \partial v > 0 \). It follows that \( dz / dv < 0 \). 

**Proof of Proposition 12:** Bank managers solve

\[
\max_{e_i, x_{ci}, x_{si}} \mathbb{E}(w_i) - \frac{1}{2} \mathbb{V}(w_i) - \frac{1}{2} \gamma_i e_i^2
\]

subject to

\[x_{ci} + x_{si} \leq 1 + L\]

and where \( \mathbb{E}(w_i) \) and \( \mathbb{V}(w_i) \) are given by (15) and (16), respectively. Since \( x_{ci} \) and \( x_{si} \) enter additively in the leverage constraint, we have

\[
\frac{\partial \left( \mathbb{E}(w_i) - \frac{1}{2} \mathbb{V}(w_i) \right)}{\partial x_{ci}} = \frac{\partial \left( \mathbb{E}(w_i) - \frac{1}{2} \mathbb{V}(w_i) \right)}{\partial x_{si}}
\]

(44)

Intuitively, if there is a constraint on the sum \( x_{ci} + x_{si} \), then the marginal utilities with respect to \( x_{ci} \) and \( x_{si} \) must be the same (zero if the constraint is not binding). From (15)–(16),

\[
\frac{\partial \left( \mathbb{E}(w_i) - \frac{1}{2} \mathbb{V}(w_i) \right)}{\partial x_{ci}} = \mu_i a_i - \left( a_i^2 x_{ci} - \psi a_i b_i x_{cj} \right)
\]

\[
\frac{\partial \left( \mathbb{E}(w_i) - \frac{1}{2} \mathbb{V}(w_i) \right)}{\partial x_{si}} = \mu_i - a_i^2 x_{si}
\]

Given (44), we get

\[
\mu_i - \left( a_i^2 x_{ci} - \psi a_i b_i x_{cj} \right) = \mu_i - a_i^2 x_{si}
\]

or simply

\[x_{ci} - x_{si} = \psi \frac{b_i}{a_i} x_{cj}\]

(45)

If the leverage constraint is binding, then

\[x_{ci} + x_{si} = 1 + L\]

(46)

Together, (45)–(46) imply

\[x_{ci}^* = \frac{1}{2} (1 + L) + \frac{1}{2} \psi \frac{b_i}{a_i} x_{cj}\]

(47)

\[x_{si}^* = \frac{1}{2} (1 + L) - \frac{1}{2} \psi \frac{b_i}{a_i} x_{cj}\]

(48)

Intuitively, portfolio allocation does not respond to \( \mu \) since the leverage constraint is binding. For future reference, notice that

\[
\frac{dx_{si}}{da_i} = -\frac{dx_{ci}}{da_i}
\]

(49)

\[
\frac{dx_{si}}{db_i} = -\frac{dx_{ci}}{db_i}
\]

(50)
The problem faced by shareholders is:

\[
\max_{k_i, a_i, b_i} \mathbb{E}(r_i - w_i)
\]

subject to

\[
\mathbb{E}(w_i) - \frac{1}{2} \mathbb{V}(w_i) - d_i(e_i) \geq u_i
\]

and that \(e_i, x_c, x_s\) belong to the best-response mappings. Let \(k_i\) be such that the constraint is exactly satisfied and rewrite the problem as:

\[
\max_{a_i, b_i} \mathbb{E}(r_i) - u_i - \frac{1}{2} \mathbb{V}(w_i) - d_i(e_i)
\] (51)

where \(\mathbb{E}(r_i)\) and \(\mathbb{V}(w_i)\) are given by (29) and (16), respectively; and \(x_{ci}, x_{si}\) by (47) and (48), respectively. The first-order conditions with respect to \(a_i\) and \(b_i\) are given by

\[
\frac{\partial}{\partial a_i} \left( \mathbb{E}(r_i) - \frac{1}{2} \mathbb{V}(w_i) \right) + \frac{\partial}{\partial x_{ci}} \left( \mathbb{E}(r_i) - \frac{1}{2} \mathbb{V}(w_i) \right) \frac{dx_{ci}}{da_i} + \frac{\partial}{\partial x_{si}} \left( \mathbb{E}(r_i) - \frac{1}{2} \mathbb{V}(w_i) \right) \frac{dx_{si}}{da_i} - \frac{\partial d_i(e_i)}{\partial a_i} = 0
\]

\[
\frac{\partial}{\partial b_i} \left( \mathbb{E}(r_i) - \frac{1}{2} \mathbb{V}(w_i) \right) + \frac{\partial}{\partial x_{ci}} \left( \mathbb{E}(r_i) - \frac{1}{2} \mathbb{V}(w_i) \right) \frac{dx_{ci}}{db_i} + \frac{\partial}{\partial x_{si}} \left( \mathbb{E}(r_i) - \frac{1}{2} \mathbb{V}(w_i) \right) \frac{dx_{si}}{db_i} = 0
\]

Given (44) and (49), this simplifies to

\[
\frac{\partial}{\partial a_i} \left( \mathbb{E}(r_i) - \frac{1}{2} \mathbb{V}(w_i) \right) - \frac{\partial d_i(e_i)}{\partial a_i} = 0
\]

\[
\frac{\partial}{\partial b_i} \left( \mathbb{E}(r_i) - \frac{1}{2} \mathbb{V}(w_i) \right) = 0
\]

From (15)–(16), we get

\[
(1 - a_i)/\gamma_i - \frac{1}{2} \left( 2a_i x_{ci}^2 - 2\psi b_i x_{ci} x_{cj} + 2a_i x_{si}^2 \right) = 0
\]

\[
-2(\psi a_i x_{ci} - b_i x_{cj}) x_{cj} + 2b_i x_{si}^2 = 0
\]

In a symmetric equilibrium

\[
(1 - a) / \gamma - ((a - \psi b) x_c^2 + a x_s^2) = 0
\] (52)

\[
-(\psi a - b) x_c^2 + b x_s^2 = 0
\] (53)

Substituting \(p\) for \(b/a\) and solving (47)–(48) for the symmetric equilibrium, we get

\[
x_c = \frac{1}{2 - \psi p} (1 + L)
\] (54)

\[
x_s = \frac{1 - \psi p}{2 - \psi p} (1 + L)
\] (55)
Substituting for $x_c$ and $x_s$ in (52)–(53) and simplifying,

\[(1 - a)/\gamma - a \left( \frac{1 - \psi p}{2 - \psi p} \right) (1 + L)^2 = 0\]
\[-(\psi - p) + p (1 - \psi p)^2 = 0\]

From the second equation, we see that $p$ is determined by $\psi$ and independent of $L$. Moreover, from the first equation,

\[
\frac{da}{dL} = - \frac{2a \left( \frac{1 - \psi p}{2 - \psi p} \right) (1 + L)}{\gamma^{-1} + \left( \frac{1 - \psi p}{2 - \psi p} \right) (1 + L)^2} < 0
\]

Finally, from (54)–(55), we get

\[
x = \frac{x_c}{x_c + x_s} = \frac{1}{2 - \psi p}
\]

The result follows. ■
References


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