Alliance Formation and Firm Value

August 2017

Abstract. We consider the formation of alliances that potentially create complementarities, that is, when the value function is super-modular in firm resources. We show that, in a frictionless world where information is perfect and managers optimize, firm alliances disproportionately increase the value of high-resource-level firms, resulting in higher variance and higher skewness of the distribution of firm value; moreover, higher-value alliances are subject to regression to the mean at a faster rate. These effects are magnified if the degree of complementarities is endogenously determined by each firm's investment. We also consider alliances where matching and/or information about firm resources are imperfect, and show that complementarities are a necessary but not sufficient condition for alliances to cause an increase in firm value; and that complementarities are neither a necessary nor a sufficient condition for alliances to be correlated with higher firm value.

Keywords: firm alliances, matching, competitive advantage.

Cabral: Paganelli-Bull Professor of Economics and International Business, Stern School of Business, New York University; and Research Fellow, CEPR; luis.cabral@nyu.edu. Pacheco-de-Almeida: Associate Professor of Strategy and Business Policy, HEC Paris; member of the CNRS unit GREGHEC, UMR CNRS 2959; gpacheco@stern.nyu.edu. While listed alphabetically, both authors contributed to the present paper and research project equally.
Financial support from the HEC Foundation (grant number 4F73DRRR - F1202 - O2200PFF1) is gratefully acknowledged. We thank Editor Olav Sorenson, as well as the Associate Editor and two anonymous reviewers for their helpful comments. We also thank Rajshree Agarwal, Denisa Mindruta, Gordon Walker, participants in the 2014 Academy of Management Meeting, and especially Michael Ryall for helpful feedback. The usual disclaimer applies.

1. Introduction

Why do some firms benefit more from alliances than others? This empirical puzzle has been a focal point of debate in the alliance literature (e.g., Kale and Singh, 2009). The classical view in corporate strategy is that alliance performance and inter-firm competitive advantage hinge on the existence of inter-firm complementarities. According to this literature, complementarities drive positive relational rents, thereby making "the whole greater than the sum of the parts" in an alliance. Much of this research goes on to identify the specific mechanisms that create and enable complementarities in firm alliances, including relationship-specific assets, knowledge-sharing routines, complementary resources and capabilities, product relatedness, absorptive capacity, and learning effects (e.g., Powell et al., 1996; Mowery et al., 1996; Dyer and Singh, 1998; Doz and Hamel, 1998; Lane and Lubatkin, 1998; Ahuja, 2000; Anand and Khanna, 2000; Kale et al., 2002). In sum, this literature suggests that the outcome of firm alliances, in terms of firm value, can be quite varied: some alliances increase firm value, while others decrease firm value.

This paper contributes to this largely empirical debate by developing a theoretical model, as well as a series of numerical simulations, that help understand the effects of alliances on the industry *distribution* of firm value. Are alliances a force toward increasing or decreasing performance differences across firms? Our results show that, even in the absence of ex-post agency and organizational problems, frictionless alliance formation amplifies the skewness of the firm value distribution. This increase in firm performance heterogeneity is due to the fact that better firms benefit disproportionately more from alliances when there are inter-firm strategic complementarities and matching is assortative. When this is the case, relational rents are a convex function of firm quality or resource levels. Moreover, these effects are magnified if the degree of complementarities is endogenously determined by each firm's investment.

We also show that, although alliances exacerbate differences in firm relative performance, they do not change firms' relative industry ranking. In other words, if firm A has a competitive advantage over firm B, alliances with other firms under frictionless matching and perfect information should not change the status quo. In a model extension, we compare the sustainability of firm performance in alliances versus standalone operations when resource value regresses to the industry mean over time. Regression to the mean is relevant for a dynamic understanding of alliance value because, over time, it offsets the increased dispersion in firm value from alliances. Our model is agnostic as to which specific mechanism — rival imitation or resource substitution (Ghemawat, 1991) — drives regression to the mean. We show that the value of an above-average firm regresses to the industry mean faster in an alliance than for standalone firms. That is, decay in resource value is disproportionately amplified by alliances when there are inter-firm strategic complementarities and matching is assortative. These findings help shed some light on the dynamics underlying sustainability of competitive advantage in a context where firm alliances are prevalent.

A byproduct of our analysis is to clarify key alliance constructs, including complementarities, synergies, relational rents, and alliance value. In our model, we follow the economics literature and equate strategic complementarities to supermodularity in the alliance value function (Amir, 2005). As in the alliance literature, synergies exist in our model when "the whole is greater than the sum of the parts," that is, when alliances yield superadditivity in firm value (Damodaran, 2005). Synergies represent the sum of relational rents in an alliance, where relational rents are given by the difference between each partner firm's value in an alliance and its value as a standalone entity (Dyer and Singh, 1998). These standard definitions give rise to a set of results concerning firm value in alliances. Notably, we show that inter-firm strategic complementarities are a necessary but not sufficient condition for alliances to exhibit synergies, generate relational rents, and increase expected firm value. On average, whether or not alliances create relational rents, synergies and boost firm value depends not only on strategic complementarities but also on the partner selection regime as well as the level of resources partner firms commit ex-post to the alliance.

Although our model is sufficiently general to be representative of different types of inter-firm partnerships, we focus on firm alliances. The model is designed at the dyadic level, but it embeds each alliance in its broader industry context by examining performance against the entire population of industry alliances. While alliances are part of a larger relational system, our paper does not consider multi-party networks of alliances. Moreover, we focus on the emergence and configuration of alliances, not on alliance governance design or ex-post integration problems.

Our main propositions hinge on the result of assortative firm matching, which is well-known in the economics literature and based on the equilibrium concept of stable matching (where no pair of players has an incentive to re-match). As explained in the model section, the assortative matching result is generally robust to different concepts of equilibrium, including the Core of the associated coalitional game, Walrasian equilibrium, or the outcome of aggregate payoff maximization.

Finally, we employ numerical simulations to scrutinize the robustness of our findings when the assumption of frictionless alliance formation is relaxed. Specifically, we consider two types of frictions. First, we study an extreme case of firm matching frictions, where firms match randomly under perfect information. Second, we examine informational frictions, that is, a case where firms match assortatively but receive an imperfect signal of their partner's resource level (e.g., a forecasting error of partner quality). In all simulations, we assume the existence of inter-firm strategic complementarities.

The numerical analyses lend credence to the robustness of the paper's main result: even with frictions, alliances increase the dispersion of the firm value distribution. Also, informational frictions are sufficient for the emergence of at least some value-destroying alliances — even when there are strategic complementarities and no matching frictions. As long as there is perfect information about partners' resource levels, alliances never decrease firm value, independently of the partner selection regime (including the extreme case of random matching). Obviously, with frictions, when industries experience widespread waves of alliances due to a "contagion effect" — that is, firms form alliances because rivals do so — there is a sharp increase in the variance of firm value and some alliances are significantly value-destroying.

In sum, our paper hones in on alliance formation — the first of three commonly accepted cornerstones of alliance success (Gulati, 1998; Kale and Singh, 2009) — to characterize how specific "birth conditions" of alliances determine firm performance and may help explain existing empirical phenomena.

■ **Brief literature review.** "Alliances present a paradox for firms. On the one hand, firms engage in a large number of alliances to secure and extend their competitive advantage and growth; on the other hand, their alliances exhibit surprisingly low success rates" (Kale and Singh, 2009: 45). This "alliance paradox" has been extensively documented in the strategy literature. According to industry reports and academic studies, strategic alliances have purportedly been growing at rates close to 25 percent annually since 1987. At the same time, alliance failure rates have been as high as 60 to 70 percent, alliance termination rates reportedly exceed 50 percent, and 30 to 70 percent of alliances do not meet their strategic and financial goals and destroy shareholder value (for a review, see Barringer and Harrison, 2000; see also Anand and Khanna, 2000; Kale et al., 2002; Doz and Hamel, 1998; Gulati and Singh, 1998; Barringer and Harrison, 2000; Oxley et al., 2009). These patterns are allegedly similar in other types of interorganizational collaborations. For example, Park and Ungson (1997) report that the dissolution rate for joint ventures is about 50 percent. The explanation often put forward for these findings is that firms tend to be overly optimistic about the benefits of interfirm collaborations and underestimate ex-post agency and organizational problems (Dyer and Singh, 1998; Kale et al., 2002).

This alliance paradox has fueled a sizable body of research that has tried to identify the key factors that lead to alliance success. In a seminal paper, Gulati (1998) categorized alliance success factors according to the stage of the alliance life-cycle: "the formation of the alliance, the choice of governance structure, [and] the dynamic evolution of alliances" — namely, ex-post management, integration, and cooperation (p. 293). The first stage — alliance formation and partner selection — has received a considerable amount of attention in the field of strategy as "one of the most influential" factors influencing alliance success (Shah and Swaminathan, 2008: 471).

A recent comprehensive survey of this alliance formation literature revealed that three main partner attributes positively affect alliance performance: (a) partner complementarity, (b) partner commitment, and (c) partner compatibility (Kale and Singh, 2009). Partner complementarity is often equated with inter-firm resource complementarities, or "the extent to which a partner firm contributes non-overlapping resources to the relationship" (Kale and Singh, 2009: 47). However, the construct of resource complementarity has been used differently by different authors. For some alliance scholars, "resource complementarity between firms [means] that ... the pooled resources can create excess value relative to their value before the pooling" (Chung, Singh, and Lee, 2000) — that is, there are alliance synergies (Damodaran, 2005) and the alliance value function is superadditive. For other scholars, in alliances "two attributes are complements ... if having more of one raises the marginal value (or the incremental return) of having more of the other" (Mindruta, 2009: 30; see also Amir, 2005; and Milgrom and Roberts, 1995) — that is, the alliance value function is supermodular. To avoid ambiguity in terminology, in this paper we define resource complementarity as supermodularity (strategic complements) and synergies as superadditivity (i.e. the whole is greater than the sum of its parts).

Besides complementarities, alliance formation and success also depends on the "partner firm be[ing] compatible with the focal firm and committed to the relationship" (Kale and Singh, 2009, 2009: 47). Partner commitment refers to the willingness of firms to make resource contributions to the alliance (Gundlach et al., 1995). Partner compatibility denotes situations in which partner firms have operations, decision-making processes, working styles and cultures that go well together (Dyer and Singh, 1998). Both partner commitment and compatibility find expression in our model but in simple reduced form, due to analytical tractability: we use an exogenous parameter that sets the level of partner resources that are considered productive or available for the purposes of the alliance. Lower commitment or compatibility cuts down the alliance resource pool in the alliance value function.

Empirical studies show that the relative importance of complementarities, commitment and compatibility varies with the alliance context. For example, Rothaermel and Boeker (2008) argue that partner complementarity is particularly relevant for alliance success if one firm is younger than its partner. Kale and Singh (2009) suggest that partner commitment is critical when alliance cooperation and implementation processes are non-contractible ex-ante or when ex-post adaptation is required in light of market or organizational uncertainty. Prior theory has postulated that repeated partner interaction should enhance firms' commitment to an alliance, working as a substitute to formal contracting; however, recent evidence has interestingly shown that this may not always be the case (Ryall and Sampson, 2009).

Finally, our paper and model abstracts away from any factor that may affect alliance performance during the two last stages of the alliance life-cycle: governance design and the issues related to the dynamic evolution of alliances (Gulati, 1998). This partial view of alliance performance helps isolate the mechanisms that we are interested in studying during alliance formation. A more detailed review of governance or ex-post management issues in the alliance literature can be found in Kale and Singh (2009).

2. Model and results

Our base model corresponds to the well-known economics model of frictionless matching with strict non-transferable utility (Chade et al., 2015).[1] The main insights in this literature date back to Gale and Shapley (1962), who show equilibrium existence, and Becker (1973), who provides conditions such that matching is assortative (a property we will also refer to as positive sorting). Our analysis does not add to the body of theoretical results, rather provides an application to the study of the impact of alliances on the distribution of firm value.[2]

1. Later in the paper we also consider the possibility of matching with frictions and/or with transferable utility.
2. Prior applications of matching theory include matching of buyers and sellers; men and women; and many others.

Given that our interest is to work with statistical distributions of firm value and alliance value, we represent an industry as a continuum of firms.[3] Each firm is endowed by Nature with a level θ of resources. We assume that $\theta > 0$ and that the value of θ is distributed according to a smooth cdf $F(\theta)$.[4]

The matching literature has for the most part considered a finite number of agents. Our existence result (Proposition 1) is also proven on a finite set. The continuum case can then be understood as a representation of the limit of the finite case as population size tends to ∞.[5,6] In our numerical simulations, we consider explicitly the more realistic case of a finite number of firms.

Much of the matching literature is based on a "marriage" framework, where elements from set A are matched with elements from a set B. Instead, we consider the "unisex" version of the model. Formally, the results are similar across both approaches (Chade et al., 2015). Moreover, although our analysis is couched in terms of firm alliances, the framework is sufficiently flexible to accommodate other applications, such as mergers and acquisitions or joint ventures. What all of these applications have in common is a situation where two firms come together in some form whereby their resources are combined to create value.

We assume that single agents (that is, firms not forming an alliance) receive a value $v_S(\theta) = \theta$, where S refers to "standalone" or "solo."[7] By contrast, if a θ-type firm and a $\widetilde{\theta}$-type firm form an alliance, then firm θ receives value $v_A(\theta, \widetilde{\theta})$, where A stands for "alliance."[8] The equilibrium concept is that of stable set of matches. Specifically, a matching outcome is stable if there exists no blocking pair of agents preferring to be matched to each other rather than to their respective partners in the candidate stable allocation (if a firm is unmatched, we count that as being matched with itself). Later we consider alternative ways of looking at the matching problem, including the Core of the associated coalitional game.

We expect $v_A(\theta, \widetilde{\theta})$ to have several properties. First, if there are no complementarities, then alliances do not increase firm value: $v_A(\theta, \widetilde{\theta}) = v_S(\theta)$. Second, the value increase from partnering with a higher-resource firm is greater the greater the resource level of the focal firm. In other words, alliances lead to complementarities across firms. Formally, this corresponds to a supermodular value function, that is, one such that $\partial^2 v_A / \partial\theta\, \partial\theta' > 0$; and if we define a parameter λ that measures the degree of complementarities, $\partial^2 v_A / \partial\theta\, \partial\theta'$ is increasing in λ. In words, $\partial^2 v_A / \partial\theta\, \partial\theta' > 0$ states that the greater θ is, the greater the gain in v_A that is derived from an increase in θ', that is, higher-θ firms benefit more from a good match than lower-θ firms; and the statement that $\partial^2 v_A / \partial\theta\, \partial\theta'$ is increasing in λ means that the previous complementarity effect is greater the greater λ is, that is, λ measures the degree of complementarity, as previously postulated.

Several mechanisms may motivate this structure of the alliance value function. For example, a partner with absorptive capacity may be able to use the focal firm's knowledge to boost its R&D productivity in a R&D alliance. The deeper the knowledge resource pool of the focal firm, the

3. Obviously, there is an approximation error when a finite set of firms is treated as a continuum. Such error may be greater when the number of firms is very small; but then the concept of firm value distribution is also less interesting.

4. In principle, it is possible for a firm to have a negative level of resources (e.g., liabilities greater than assets). However, we would expect this to be a temporary situation; and accordingly assume that $\theta > 0$.

5. Specifically, consider a set of n firms obtained from $F(\theta)$ by sampling n times independently. Our equilibrium concept applies to such finite set. However, to the extent that the size of the finite set is very large, we directly work with a continuum of types with a distribution $F(\theta)$. In fact, as $n \to \infty$ the observed population distribution converges almost surely to $F(\theta)$.

6. Gretsky et al. (1992) prove that the continuous model is the limit of Shapley and Shubik's (1972) finite-agent assignment model. We are not aware of an equivalent result for the unisex matching case, which is the one we consider.

7. We assume that a firm's resources can be measured by a one-dimensional variable; and that the mapping from resources to value is linear. Given this, with no further loss of generality we assume the identity map (in other words, we can always change units of resource or value measurement so that the linear map holds. Later we consider the possibility of a firm's resources be a multi-dimensional vector.

8. One important difference between transferable and non-transferable utility is that, in the former, value functions define the total value created by a match and the precise value captured by each of the players results from some equilibrium notion. By contrast, under non-transferable utility, there is an exogenously-given function that determines the value each player gets from a given match.

stronger this effect. Alternatively, if the focal firm invests in relationship-specific assets, the partner firm may also be able to contribute more to the alliance: site-specific investments may reduce transportation and coordination costs to the partner; physical-asset specificity such as investments in customized machinery may allow greater product differentiation and quality for the same investments by partner firms; and human co-specialization has been shown to enhance partner quality and speed-to-market (Dyer and Singh, 1998).

Based on these considerations, we assume the following functional form:

$$v_A(\theta, \widetilde{\theta}) = \theta + \lambda\,(\widetilde{\theta} - \alpha)\,\theta \tag{1}$$

where α is a parameter measuring the lowest partner resource level such that an alliance increases firm value. Specifically, the value of α may be interpreted as an indicator of the amount of resources that the partner firm keeps away from the alliance.[9] Alternatively — and perhaps even better — α captures the minimal resource commitment necessary for a successful alliance.[10] Empirical evidence suggest that one of the predictive factors of success in a strategic alliance is the partners' commitment to the relationship, that is, their willingness to commit time, money and facilities to the relationship (Gundlach et al., 1995).[11] Although for most of the paper we take α as an exogenous parameter, its value has a natural behavioral interpretation as the level of firm-$\widetilde{\theta}$ resources that are kept unavailable for the purposes of the alliance, that is, an inverse indicator of firm $\widetilde{\theta}$'s commitment to the relationship. If firm $\widetilde{\theta}$ is not very cooperative and keeps a high value α of its resources away from firm θ, then even an alliance with a high-$\widetilde{\theta}$ firm will be of little use for the focal θ firm.

Dyer and Singh (1998) define relational rent as the difference between the firm's value in an alliance and the value it would attain had it not entered into an alliance. In the present context, the relational rent earned by a θ-type firm that forms an alliance with a $\widetilde{\theta}$-type firm is given by $\lambda\,(\widetilde{\theta} - \alpha)\,\theta$.

Suppose that $\lambda = 0$. Then there are no complementarities and $v_A(\theta, \widetilde{\theta}) = v_S(\theta)$. Suppose that $\lambda > 0$ and that $\theta, \widetilde{\theta} > \alpha$. Then $v_A(\theta, \widetilde{\theta}) + v_A(\widetilde{\theta}, \theta) > v_S(\theta) + v_S(\widetilde{\theta})$, that is, the alliance implies positive synergies, which we define as the value difference between the whole (alliance) and the sum of the parts (standalone values).[12] Moreover, the synergy implied by the alliance,

$$\varsigma \equiv \Big(v_A(\theta, \widetilde{\theta}) + v_A(\widetilde{\theta}, \theta)\Big) - \Big(v_S(\theta) + v_S(\widetilde{\theta})\Big) = \lambda\,(\theta - \alpha)\,\widetilde{\theta} + \lambda\,(\widetilde{\theta} - \alpha)\,\theta$$

is increasing in λ. For these reasons, even though, strictly speaking, λ is a measure of complementarities (not synergies), we note that, to the extent that $\theta, \widetilde{\theta} > \alpha$, λ is an indicator of the degree of synergies as well. However, while synergies are a dyad-specific construct because they vary across alliances with both partners' resource levels, complementarities are alliance invariant. Note also that an alliance's synergy is equal to the sum of the relational rents of the two participating firms, so λ is also an indicator of the size of the relational rents (for given values of θ and $\widetilde{\theta}$).

Some additional notes regarding the expression for $v_A(\theta, \widetilde{\theta})$ are in order. First, the effect of an alliance on firm value may be positive or negative. In particular, if firms' types are sufficiently low ($\theta, \widetilde{\theta} < \alpha$) then an alliance *decreases* joint firm value. If $\alpha = 0$ ("full commitment"), however, then all alliances increase firm value. Second, (1) has the property that the alliance between two firms with different resource levels benefits the lower-θ firm more than it benefits the higher-θ firm. In

9. In order to be consistent wth the analysis that follows, it must be that $\alpha \in [0, \overline{\alpha}]$, where $\overline{\alpha} > \widetilde{\theta}$. In other words, if the partner firm is sufficiently uncommitted, then $\widetilde{\theta} < \alpha$ and the alliance is value destroying (from firm θ's point of view).

10. We do not explicitly consider the cost of entering into an alliance. Adding a cost parameter create would create an additional degree of freedom, which would weaken the results without adding new results. That said, one can think of α as reflecting the opportunity cost of entering into an alliance.

11. Another important factor in the alliances literature is resource compatibility (Dyer and Singh, 1998). Our assumption that a firm is wholly characterized by a one-dimensional value θ effectively abstracts from these issues.

12. Damodaran (2005) defines synergy as "the additional value that is generated by combining two firms, creating opportunities that would not been available to these firms operating independently."

fact, Firm 1's increase in value is given by $\lambda\,(\theta_2 - \alpha)\,\theta_1$, whereas Firm 2's increase in value is given by $\lambda\,(\theta_1 - \alpha)\,\theta_2$; and if $\theta_1 > \theta_2$ then $\lambda\,(\theta_1 - \alpha)\,\theta_2 > \lambda\,(\theta_2 - \alpha)\,\theta_1$.

■ **Modeling choices.** Our analysis is based on a matching model with non-transferable utility, by which we mean that a player's payoff from a given match is exogenously determined by $v_{\scriptscriptstyle A}(\theta,\widetilde{\theta})$. Moreover, we consider the equilibrium concept of stable matching, by which we mean that no pair of players have an incentive to re-match.

The choice of a non-transferable-utility game has the benefit of starting from a specific payoff function that obeys certain desirable rules, namely supermodularity. This allows us to derive the distribution of firm value resulting from a given alliance pattern. By contrast, generically the Core — a common equilibrium concept in games of this sort — does not pin down a specific equilibrium payoff, which complicates the task of deriving the firm value distribution.

Which approach is more realistic: transferable or non-transferable utility? Models with transferable utility assume that agents (firms in our case) can make side payments, which we do observe in some contexts (e.g., determining the conversion rate in a stock-for-stock merger). Alliances and other arms-length agreements may fit the non-transferable-utility assumption better, as side payments may be difficult (e.g., due to moral hazard problems) or simply illegal. Reality is probably somewhere between the extremes of transferable and non-transferable utility, and the extreme cases may be interpreted as reference benchmarks.

Fortunately, the choice of model and equilibrium concept are not crucial in terms of our main results. First, we provide conditions such that alliances follow a positive sorting pattern; and as shown by Chade et al. (2015), positive sorting is a very robust result: it takes place both with transferable and with non-transferable utility; and it results from various equilibrium concepts, including welfare maximization, Walrasian equilibrium, stability, and the Core. In particular, the result of positive sorting in equilibrium obtains in the Core of the coalitional game associated to the present matching problem. The coalitional (a.k.a. cooperative) game in question would have the following characteristic function: $v(\{\theta\}) = v_{\scriptscriptstyle S}(\theta)$; $v(\{\theta,\widetilde{\theta}\}) = v_{\scriptscriptstyle A}(\theta,\widetilde{\theta}) + v_{\scriptscriptstyle A}(\widetilde{\theta},\theta)$; and $v(\varsigma) = 0$ for any other subset ς of the set of players.

Second, we derive the impact of alliances on the firm value distribution. However, to the extent that matching is assortative, our payoffs would also result from a symmetric equilibrium with side payments. Specifically, in a symmetric equilibrium with positive sorting the value of an alliance between two θ-type firms is equally split between the parties, both in the equilibrium with transferrable utility and in the equilibrium with no transferrable utility.[13]

Throughout the paper, we assume that a firm's resources can be summarized by a one-dimensional variable θ. In reality, we would expect firm resources to include several dimensions. Just as in consumer theory or producer theory, the one-dimensional characterization is correct under one of two extreme assumptions: perfect correlation across the various dimensions; or perfect substitutability across the various dimensions. If a firm that has more of θ_1 also has more of θ_2, and in the same amount, then θ_1 provides a sufficient statistic of the firm's resource level; that is, the analysis can be done solely based on the value of θ_1. At the opposite extreme, if resources are perfect substitutes, then we can think of θ as a composite of the firm's resource levels:

$$\theta = \sum_{i=1}^{n} \omega_i\,\theta_i$$

where ω_i/ω_j measures the marginal rate of substitution between resources i and j, which we assume is constant.[14]

The alliance value function (1) makes one implicit assumption: the degree of complementarities, λ, is not a function of firm type or the particular match in question. We make this assumption

13. In this sense, our analysis has relatively little to say about the process of value capture (Gans and Ryall, 2016).

14. In the economics literature on consumer demand with differentiated products, this corresponds to the hedonic-prices approach (Bajari and Benkard, 2005).

for analytical simplicity and as a benchmark that helps highlight the effect of matching patterns on the distribution of firm value. Later in this section we do allow for the possibility of λ being endogenously determined; that is, we add a preliminary stage in which firms can choose (at a cost) the value of their λ.

We note that, in addition to frictionless assortative matching, we will also consider alternative alliance patterns, including random matching and matching based on noisy signals. The purpose of these alternative scenarios is better to understand the relative role played by positive sorting, imperfect information and non-optimizing firm behavior in terms of the firm value distribution. Specifically, imperfect information is modeled by assuming that firms observe a noisy signal of θ, namely $\theta + \epsilon$, where ϵ is a zero-mean random observational shock; and by assuming that stability applies to expected value from a match. An extreme version of imperfect information is random matching. This corresponds to the limit when the variance of ϵ goes to infinity, so that matching is based on an uninformative signal. Non-optimizing behavior is modeled by assuming that some firms form an alliance even though relational rents are negative, that is, firms would be better off by remaining as a standalone organization.[15]

■ **Preliminary results.** We begin by presenting some results regarding the alliance value function and its implications. The first two are not propositions in the proper sense of the word, to the extent that they result from our assumptions regarding the alliance value function. We should therefore think about them as *properties* of the value function. The third result is an important result from the matching literature which we will use as a building block for some of our later propositions.

Property 1. *If $\lambda = 0$, then firm value is independent of the pattern of firm alliances.*

The proof is trivial: if $\lambda = 0$, then $v_A(\theta, \theta') = \theta = v_S(\theta)$, that is, firm value remain unchanged regardless of the alliance partner's type. In words, complementarities are a necessary condition for alliances to have a positive impact on firm value. Absent those complementarities, firm alliances only have a "cosmetic" effect, that is, have no effect on the fundamentals of firm value.

Next we show the impact of alliances on firm value depends crucially on the value of α. Let $\mathbb{E}(x)$ be the expected value of x.

Property 2. *Suppose that all firms are randomly matched and that all pairs form an alliance. Then on average firm value increases if and only if $\mathbb{E}(\theta) > \alpha$. In particular, if $\alpha = \mathbb{E}(\theta)$ then firm value remains constant regardless of the degree of complementarities (λ).*

The proof is, again, a one-liner: the expected value for a θ firm upon forming an alliance with a randomly selected partner is given by

$$\mathbb{E}_{\widetilde{\theta}}\big(v_A(\theta, \widetilde{\theta})\big) = \theta + \lambda \left(\mathbb{E}_{\widetilde{\theta}}(\widetilde{\theta}) - \alpha\right) \theta = \theta \left(1 + \lambda \left(\mathbb{E}(\theta) - \alpha\right)\right)$$

where $\mathbb{E}_x$ denotes the expected-value operator with respect to variable x. This is greater than θ if and only if $\mathbb{E}(\theta) > \alpha$; and is independent of λ if $\mathbb{E}(\theta) = \alpha$.

As mentioned earlier, α can be interpreted as the level of resources *not* committed to an alliance. Property 2 can thus be restated as follows: given random matching, the average impact of an alliance on firm value is positive if and only the level of commitment by alliance members is sufficiently high.

We next come to the our central result regarding the pattern of alliance formation. This is not a novel result. Our contribution will be its application in the particular context of firm alliances. Recall that we model the industry as a continuum for easier handling of distributions. Our equilibrium result is based on a finite number of firms with different values θ. In this sense, the idea of a θ firm being matched with a θ firm should be interpreted as the limit of a match with a firm with similar θ as the number of firms becomes large.

15. Later in the paper we discuss reasons why firms might behave non-optimally when it comes to alliance formation.

Proposition 1. *Suppose there is a finite number of firms, ordered by their value of θ: $\theta_1 > \theta_2 > \ldots$. In equilibrium (which is unique), θ_1 is matched with θ_2, θ_3 with θ_4, and so on, so long as $\theta_i > \alpha$; and all firms with $\theta_i < \alpha$ remain as standalone organizations.*

Proof: We adapt the results in Gale and Shapley (1962) and Becker (1973) to the case of unisex matching. Consider the following series of matches:

$$\mathcal{M} = \{(1,2), (3,4), \ldots\}$$

We next argue that this is the unique stable set of matches. Suppose that, differently from $\mathcal{M}$, firm i is matched with j such that $j > i + 1$. Then there exists another pair (k, l) such that $i > k > j$. We then have four possibilities regarding relative rankings:

$$\begin{aligned} i < k < j < l \\ i < k < l < j \\ i < l < k < j \\ l < i < k < j \end{aligned}$$

In all cases, it is possible to rematch these two pairs so as to increase total payoff. For example, in the first case the alternative matches (i, k) and (j, l) lead to higher payoff:

$$\begin{aligned} v_A(\theta_i, \theta_k) + v_A(\theta_j, \theta_l) &= \theta_i + \lambda\,(\theta_k - \alpha)\,\theta_i + \theta_k + \lambda\,(\theta_i - \alpha)\,\theta_k \\ &+ \theta_j + \lambda\,(\theta_l - \alpha)\,\theta_j + \theta_l + \lambda\,(\theta_j - \alpha)\,\theta_l \\ &= \sum \theta_t - \lambda\,\alpha \sum \theta_t + 2\,\lambda\big(\theta_i\,\theta_k + \theta_j\,\theta_l\big) \\ &> \sum \theta_t - \lambda\,\alpha \sum \theta_t + 2\,\lambda\big(\theta_i\,\theta_j + \theta_k\,\theta_l\big) \\ &= v_A(\theta_i, \theta_j) + v_A(\theta_k, \theta_l) \end{aligned}$$

where the strict inequality follows from strict supermodularity. In fact,

$$\big(\theta_i\,\theta_k + \theta_j\,\theta_l\big) = \big(\theta_i\,\theta_j + \theta_k\,\theta_l\big) + \big(\theta_k - \theta_j\big)\big(\theta_i - \theta_l\big)$$

and

$$\big(\theta_k - \theta_j\big)\big(\theta_i - \theta_l\big) > 0$$

follows the assumption that $\theta_i > \theta_k > \theta_j > \theta_l$. A similar argument shows that there is no deviation from $\mathcal{M}$ that increases both partners' payoff. We conclude that $\mathcal{M}$ is the unique stable set of matches. ■

In words, positive sorting implies that the two highest-θ firms form an alliance, then the next two highest, and so forth. As mentioned earlier, for the purposes of studying the distributional implications of firm alliances, we work with a continuum of firms. The natural way of linking this to our finite-case result (as well as the finite-set simulations we perform below) is to consider a set n of firms drawn independently from $F(\theta)$. From that set, we construct a ranking such that $\theta_i > \theta_j$ if and only if $i < j$.[16] As per Proposition 1, a firm gets matched with the next firm in the ranking. As the value of n tends to infinity, a θ firm is matched almost surely with another firm with θ' arbitrarily close to θ. In terms of the continuum representation, we say that each type θ is matched with a type θ.[17]

■ **Alliances and performance.** We now present a set of results regarding firm alliances and firm value. The continuum counterpart of our finite-firm case consists of assortative matching by firms

16. Since $\theta_i = \theta_j$ with probability zero, generically we can construct such ranking of strict inequalities.

17. In other words, if $f(\theta)$ is the density of firms of type θ, then a measure $\frac{1}{2}\,f(\theta)$ gets matched with a measure $\frac{1}{2}\,f(\theta)$.

with $\theta > \alpha$. In other words, firms with high resource level ($\theta > \alpha$) are matched with firms of equal resource level; and firms with low resource level ($\theta < \alpha$) remain as standalone organizations. In this setting, no firm lowers its value as a result of alliance formation.

In practice (and as empirical evidence suggests) an important fraction of alliances decrease firm value. Under frictionless matching, this implies non-optimizing behavior on the part of some of the firms engaged in alliances. Although our model is static, one explanation for value-decreasing alliances is that firms with $\theta < \alpha$ imitate firms with $\theta > \alpha$ and are led into alliances with negative relational rents.

The literature on alliance waves (Gomes-Casseres, 2004) suggests a variety of reasons why imitation may take place. One is vicarious learning (Bikhchandani et al., 1992; Banerjee, 1992), which may lead firms into the fallacy that "what's good for others is good for me." A second cause is given by management "fashion" or "fads" or "peer pressure," whereby firms imitate each other for socio-psychological motives (Abrahamson, 1996).

Still another explanation for value-decreasing alliances is the existence of frictions, in particular imperfect information. We will take up this issue in the next section. For the time being, we assume that all firms with θ greater than a certain threshold $\theta_\circ$ form an alliance. In the equilibrium of the finite-firm case, as stated by Proposition 1, such threshold is given by α. We model non-optimizing behavior by considering threshold values $\theta_\circ < \alpha$, including the limit case $\theta_\circ = 0$ (whereby all firms form an alliance).

Proposition 2. *Suppose that $\lambda = 0$. Suppose moreover that firms form an alliance (with an equal-θ firm) if and only if $\theta > \theta_\circ$, where $\theta_\circ$ is a particular threshold value of θ. Then firms that form an alliance have higher value than firms that do not. However, average firm value is the same before and after alliances take place.*

Proof: Let $\overline{v}_1$ be average firm value of firms that form alliances; and $\overline{v}_0$ the average firm value of firms that do not form alliances. Then

$$v_1 = \mathbb{E}\left[\theta \,|\, \theta > \theta_\circ\right] > \mathbb{E}\left[\theta \,|\, \theta < \theta_\circ\right] = v_0$$

where $\mathbb{E}\left[\theta \,|\, \theta > \theta_\circ\right]$ is the expected value of θ conditional on θ being greater than the threshold $\theta_\circ$. ■

Proposition 2 reflects the classic selection problem and the pitfalls of confusing correlation with causality. If we do not control for selection, then a regression of firm value with alliance membership on the right-hand side may produce a positive coefficient even though there is no causal effect from being in a alliance to increasing firm value. These types of regressions were relatively common in the early strategy literature on alliances. Since then, the field has evolved and now rightly acknowledges and accommodates this empirical problem.

The issue of selection is important from a methodological point of view, but also from a behavioral point of view. As we will see in the next section, the degree to which firms correctly select into forming an alliance has important implications in terms of industry firm value distribution.

We next assume that there are complementaries and inquire whether in this context alliances cause an increase in firm value.

Proposition 3. *Suppose that firms are assortatively matched (each θ type with another θ type); and that all pairs form an alliance. A θ firm increases value if and only if $\theta > \alpha$. Moreover, if $\mathbb{E}(\theta) \geq \alpha$ then on average alliances strictly increase firm value.*

Proof: Assortative matching implies that $v_A(\theta, \theta) = \theta + \lambda\,(\theta - \alpha)$, which is greater than $v_S(\theta) = \theta$ if and only if $\theta > \alpha$. If $\alpha = \mathbb{E}(\theta)$, then the increment in average firm value resulting from firm alliances is given by

$$\int_{-\infty}^{+\infty} v_A(\theta, \theta) - v_S(\theta)\; dF(\theta) = \int_{-\infty}^{+\infty} \lambda\left(\theta - \mathbb{E}(\theta)\right)\, \theta\; dF(\theta) > \lambda\left(\mathbb{E}(\theta) - \mathbb{E}(\theta)\right)\, \mathbb{E}(\theta) = 0$$

where the inequality follows from Jensen's inequality (e.g., Royden, 1968: p. 110) and the fact that $\lambda\left(\theta - \mathbb{E}(\theta)\right)\theta$ is a convex function of θ. Finally, lower values of α lead to higher increases in firm value, so the result is strengthened. ■

Proposition 3 shows the power of complementarities: even when $\mathbb{E}(\theta) = \alpha$ and *all* firms form an alliance with their equal, on average firm value increases. Specifically, suppose — as we will do in the next section — that θ is normally distributed. If $\mathbb{E}(\theta) = \alpha$, then alliances increase the value of exactly one half of the firms and decrease the value of exactly one half of the firms. However, on average, firm value increases.

Together, Property 2, Proposition 2 and Proposition 3 imply the following result:

Corollary 1. *Complementarities are a necessary but not sufficient condition for a positive effect of firm alliances.*

If complementarities — measured by λ — are inexistent, then the effect of firm alliances is zero, regardless of how many alliances are formed or the process by which they are formed (Property 2, Proposition 2). Conversely, if complementarities are positive and if all firms form an alliance, then the average impact of alliances is negative if $\mathbb{E}(\theta) < \alpha$ and matching is random (Property 2) or matching is assortative but α sufficiently hight (Proposition 3). In other words, Corollary 1 states that complementarities are only sufficient if firms match optimally and commit to the alliance.

■ **Alliances and relative performance.** The previous set of results relates to the effect of firm alliances on absolute firm value. We now consider the companion question of the effect of alliances on firm relative performance. Does the process of firm alliances benefit some firms more than others? We answer in the affirmative.

As a preliminary result, we look at the distribution of relational rents. Following Dyer and Singh (1998), we define relational rent (under assortative matching) as the difference between the firm's value in an alliance (with a partner with the same resource level) and the value it would attain had it not entered into an alliance, assuming assortative matching, that is, $\widetilde{\theta} = \theta$, and assuming that the alliance takes place in equilibrium (that is, it increases value):[18]

$$r(\theta) \equiv v_A(\theta, \theta) - v_S(\theta)$$

Proposition 4. *Suppose firms are assortatively matched. Then relational rents are increasing in θ.*

Proof: Substituting the values of v_S and v_A, relational rents are given by

$$r(\theta) \equiv v_A(\theta, \theta) - v_S(\theta) = \theta + \lambda\,(\theta - \alpha)\,\theta - \theta = \lambda\,(\theta - \alpha)\,\theta$$

which is increasing in θ if $\theta > \alpha$. ■

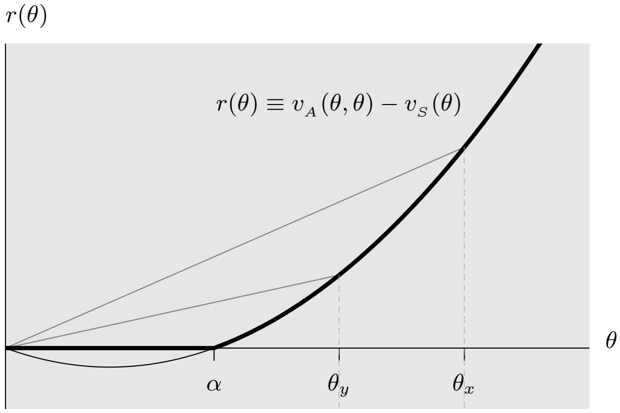
We thus conclude that $r(\theta)$ is strictly increasing in θ whenever $\theta > \alpha$ (the condition for an alliance to actually take place). Intuitively, because of the nature of complementarities, namely the fact that $\partial^2 v_A / \partial\theta\ \partial\theta' > 0$, higher-$\theta$ firms benefit more from alliances than lower-θ firms (that is, assuming assortative matching).

Our next result is similar: it states that the relative gap between high- and low-resource-level firms increases as a result of assortative-matching alliances.

Proposition 5. *Suppose firms are assortatively matched and that $\theta_x > \theta_y > \alpha$. Then $v_A(\theta_x, \theta_x)/v_A(\theta_y, \theta_y) > v_S(\theta_x)/v_S(\theta_y)$*

18. Dyer and Singh (1998) define relational rents as "a supernormal profit jointly generated in an exchange relationship that cannot be generated by either firm in isolation and can only be created through the joint idiosyncratic contributions of the specific alliance partners."

Figure 1
Assortative matching and alliance value



Proof: From (1), $v_A(\theta,\theta)/v_S(\theta) = 1 + \lambda(\theta - \alpha)$. The results follows. ■

In words, Proposition 5 establishes the contrast between the absolute and the relative effects of firm alliances. In absolute terms all firms with θ above α benefit from forming an alliance. Moreover, due to assortative matching the relative positioning of firms remains constant as the result of firm alliances. Proposition 5 implies that firm θ_y is worse off in relative terms as the result of the process of firm alliances, that is, it improves proportionately less than firm x.

Figure 1 illustrates Propositions 4 and 5. The quadratic line $\theta(\theta - \alpha)$ measures the potential relational rent in case a type θ firm engages in an alliance with another firm of the same type. Such an alliance only takes place if the relational rent generated is positive. We thus conclude that under assortative matching the relational rent is given by the maximum of zero and $\theta(\theta - \alpha)$, the thick line in Figure 1. Clearly, the relational rent is increasing in θ, strictly increasing if $\theta > \alpha$.

Regarding Proposition 5, notice that, if $\theta > \alpha$, then the ratio $r(\theta)/\theta$ is strictly increasing in θ. This can be seen from the slope of the line that goes from the origin to the point $(\theta, r(\theta))$. As shown earlier, this implies that the ratio $v_A(\theta_x,\theta_x)/v_A(\theta_y,\theta_y)$ is greater than the ratio $v_S(\theta_x)/v_S(\theta_y)$. In other words, the process of assortative matching increases the gap between firms of different resource levels.

We have just established that high-θ firms benefit from alliances more than low-θ firms. What implications does this have for the distribution of firm value? In the appendix we prove two results that address this question. First, we show (quite generally) that, in equilibrium, the skewness of the distribution of firm value increases after alliances have taken place. Intuitively, supermodularity of the matching function and assortative matching together imply that high-θ firms benefit from alliances more than low-θ firms. This in turn implies that the right tail of the distribution of firm value becomes thicker, which, in statistical terms, is captured by the measure of skewness.

Second, we provide sufficient conditions such that the variance of firm value increases after alliances have taken place; and moreover the increase in variance is increasing in λ. Broadly speaking, the intuition is the same as for the skewness result: under assortative matching, the best get matched with the best and become even better, thus increasing the thickness of the right tail.[19]

■ **Imperfect information.** Our analysis so far assumes that the values of $\theta, \widetilde{\theta}$ are perfectly observed. This is a useful benchmark but obviously not a very realistic assumption. Suppose now that the values of each firm's resources are observed with noise. Specifically, all firms observe a signal of a

19. Our variance result is more restrictive because, in addition to thickening the right tail, alliances also thin out the left tail. This second effect reinforces the increase in skewness but may work against increasing variance.

θ-type firm's resource level that is given by

$$\theta' = \theta + \epsilon \tag{2}$$

where ϵ is an independently and identically distributed random observational shock with zero mean and cdf $G(\epsilon)$.[20] The natural extension of the equilibrium concept considered above is stable matching in expected value. Specifically, a set of matchings is stable if and only if no pair of firms can increase their joint *expected* payoff by rematching away from the proposed equilibrium.

Given an observed signal θ' and the distribution $G(\epsilon)$, the posterior on the true value of θ is given by $G_\theta(\theta \mid \theta')$. Let μ be its mean value; and let $\widetilde{\mu}$ be the mean value corresponding to $G_{\widetilde{\theta}}(\widetilde{\theta} \mid \widetilde{\theta}')$. As in Proposition 1, we derive equilibrium results for the finite case (of which the continuum case is a representation). Our main result is that, in the equilibrium of the corresponding finite-agent model, matching is assortative in terms of the observed signals; and that expected firm value from alliances is lower than under perfect information.

Proposition 6. *Suppose there is a finite number of firms. Suppose that the values of θ are observed imperfectly: $\theta' = \theta + \epsilon$. In equilibrium, firms are assortatively matched according to the observed signal θ' so long as $\theta' > \alpha$; wheres all firms with $\theta' < \alpha$ remain as standalone organizations. Finally, expected firm value is lower than under perfect information.*

Proof: The expected value of a given match is given by

$$\begin{aligned}
&\mathbb{E}\Big(v_A(\theta,\widetilde{\theta}) + v_A(\widetilde{\theta},\theta) \mid \theta', \widetilde{\theta}'\Big) = \\
&= \int_{\theta'} \int_{\widetilde{\theta}'} \Big(\theta + \lambda\,(\widetilde{\theta} - \alpha)\,\theta + \widetilde{\theta} + \lambda\,(\theta - \alpha)\,\widetilde{\theta}\Big)\; dG_\theta(\theta \mid \theta')\; dG_{\widetilde{\theta}}\Big(\widetilde{\theta} \mid \widetilde{\theta}'\Big) \\
&= \mu + \lambda\,(\widetilde{\mu} - \alpha)\,\mu + \widetilde{\mu} + \lambda\,(\mu - \alpha)\,\widetilde{\mu} \\
&= v_A(\mu,\widetilde{\mu}) + v_A(\widetilde{\mu},\mu)
\end{aligned}$$

It follows that Proposition 1 applies, with the proviso that we substitute expected values for actual values.

Chade et al. (2015) show that, under frictionless matching, the stable matching solution corresponds to the total value maximizing solution. Therefore, expected payoff under imperfect information must be lower than under perfect information. ■

There are two reasons why imperfect information leads to lower firm value. First, some firms form alliances that lead to lower value, that is, alliances with negative relational rents. This is the case when the partner is observed to have $\widetilde{\mu} > \alpha$ but actually has $\widetilde{\theta} < \alpha$. Second, even when $\theta, \widetilde{\theta} > \alpha$, imperfect information leads to imperfect matching, that is, matching that is assortative but less then perfectly assortative; and this too leads to lower overall firm value. In the next section we return to this issue.

■ **Endogenous complementarity factor λ.** For most of the paper we have treated λ as a parameter measuring complementarities: how much a θ-type firm benefits from interacting from a $\widetilde{\theta}$-type firm. We did not delve much into the micro-foundations of the value of λ, which measures the intensity of this interaction. As mentioned in the introduction, the sources of complementarities in firm alliances include relationship-specific assets, knowledge-sharing routines, complementary resources and capabilities, product relatedness, and absorptive capacity (Dyer and Singh, 1998; Harrigan, 1988; Mowery et al., 1996; Doz and Hamel, 1998; Lane and Lubatkin, 1998; Ahuja, 2000; Kale et al., 2002).

For example, a firm with higher absorptive capacity benefits more from inter-firm knowledge spillovers from its alliance partner, which increases its own marginal resource productivity in the

20. We assume all firms observe each firm's signal θ', that is, we assume away information asymmetries.

alliance, or inter-firm complementarities. Lin et al. (2012) suggest that the value of an alliance is positively related to the level of absorptive capacity, which is consistent with (1). Moreover, in their seminal paper, Cohen and Levinthal (1990) argue that "R&D contributes to a firm's absorptive capacity." This is consistent with our model extension underlying Proposition 7, whereby λ can be chosen at a cost.

Accordingly, we assume that the degree of complementarities λ is endogenously chosen by the firm at a cost, which we assume is given by $\lambda^2/2$.[21] We now consider a game where each firm chooses the value of λ, paying the cost $\lambda^2/2$ for such investment; and then firms are assortatively matched (as before). Our next result shows that the possibility of endogenously choosing the value of λ magnifies the skewness-increasing effect of firm alliances:

Proposition 7. *Suppose that all firms are assortatively matched and form an alliance if and only if $\theta > \alpha$. The skewness of the firm value distribution is greater when λ is endogenously chosen than when it is exogenously given.*

Proof: A type θ firm that anticipates forming an alliance with a type θ firm anticipates a payoff of

$$\lambda\,(\theta - \alpha)\,\theta$$

The firm therefore maximizes

$$\lambda\,(\theta - \alpha)\,\theta - \tfrac{1}{2}\,\lambda^2$$

The optimal level of λ is given by

$$\lambda^*(\theta) = (\theta - \alpha)\,\theta$$

resulting in an overall payoff of

$$v(\theta) = \lambda^*\,(\theta - \alpha)\,\theta - \tfrac{1}{2}\,(\lambda^*)^2 = \tfrac{1}{2}\big((\theta - \alpha)\,\theta\big)^2$$

If λ is exogenously given, the firm value is simply given by $v^\circ(\theta) = \lambda\,(\theta - \alpha)\,\theta$. It follows that $v(\theta)$ is a convex transformation of $v^\circ(\theta)$:

$$v(\theta) = \left(\frac{v^\circ(\theta)}{4\,\lambda}\right)^2$$

van Zwet (1964) proves the following result: if $y = \phi(x)$ is convex and strictly increasing, then all odd standardized central moments of y are higher than the corresponding moments of x. It follows that the distribution of firm value when λ is endogenous has greater skewness. ■

Intuitively, the idea is that, the better a firm's type, the more it has to gain from an alliance with an equal. Specifically, the better a firm's type, the more it will invest in increasing the value of λ, which magnifies the effect of endogenous matching.

To put it differently, relational rents — defined in a broader way that allows for non-assortative matching — are now given by

$$r(\theta, \widetilde{\theta}) = \lambda(\theta)\big(\widetilde{\theta} - \alpha\big)\,\theta$$

A higher θ firm now receives higher relational rents for three reasons: First, it has higher resources, which are one of the "inputs" into the complementarity effect of an alliance (e.g., due to absorptive capacity). Second, it gets matched with a high-resource firm, specifically one with the same resource level θ. Third — and this is the novel effect considered in Proposition 7 — it will have made a greater pre-alliance investment in anticipation of the complementarities to be gained, leading to a higher $\lambda(\theta)$.

21. Specifically, we assume the cost function is quadratic. The coefficient $\frac{1}{2}$ multiplying λ^2 is assumed for convenience and with no further loss of generality.
It may seem a bit contrived for firms to invest in complementarities. We believe that similar qualitative results would be obtained if we considered instead that firms invest in their own resource level.

■ **Alliances and regression to the mean.** So far we have considered what is essentially a static analysis: firms are endowed with a certain level of resources, θ, and seek a partner firm to form an alliance with. We now consider a possible dynamic extension of our model.

An extensive empirical literature in economics and strategy shows that there is some persistence in firm performance, but that above-average profits tend to revert to the industry mean over time. Regression to the mean is relevant for a dynamic understanding of alliance value because it may potentially offset over time our main result that alliances increase the dispersion in the distribution of firm value. Statistically, regression to the mean is typically described by auto-regressive models of firm performance and value (Mueller, 1986; Pakes, 1987; Ghemawat, 1991; Waring, 1996). Accordingly, we assume that the value of θ follows an AR(1) process:

$$\theta_t = (1-\rho)\,\overline{\theta} + \rho\,\theta_{t-1} + \epsilon_t \tag{3}$$

where $\overline{\theta}$ is a parameter (the long-run average of θ); $\rho \in (0,1)$ is a parameter measuring serial correlation; and the error term ϵ_t has zero mean, a cdf $G(\epsilon)$, and is independent across firms and time.

In the absence of firm alliances, firm value is simply given by θ_t at time t. We thus have a dynamic process with regression to the mean. Specifically, the expected change from period t to period $t+1$ is given by

$$\mathbb{E}_t(\theta_{t+1}-\theta_t) = (1-\rho)\,\mathbb{E}(\theta) + \rho\,\theta_t - \theta_t = -(1-\rho)\,(\theta_t - \overline{\theta})$$

where $\mathbb{E}_t$ denotes expectation conditional on information available at time t. In words, if a firm has above average value, then we expect its value to decline by $(1-\rho)\,(\theta_t-\overline{\theta})$.

Consider now an industry where alliances take place and where each firm's resources evolve according to (3). Does the value of firms involved in alliances also regress to the mean? If so, how fast? The next result provides an answer to these questions.

Proposition 8. *Suppose firms are assortatively matched. The value of an above-average firm that forms an alliance at time t and keeps it until time $t+1$ regresses to the mean faster than it would had the firm not formed an alliance. Moreover, the speed of regression to the mean is increasing in λ.*

Proof: As shown in the text (Equation 3), a firm's resource level at $t+1$ is given by

$$\theta_{t+1} = (1-\rho)\,\overline{\theta} + \rho\,\theta_t + \epsilon_{t+1}$$

A firm that remains as a standalone operation has value

$$v_t = \theta_t$$

and thus expects

$$\mathbb{E}_t(v_{t+1}-v_t) = (1-\rho)\,\overline{\theta} + \rho\,\theta_t - \theta_t = -(1-\rho)\,(\theta_t-\overline{\theta})$$

If instead firm θ is in alliance with firm $\widetilde{\theta}$, then firm value is given by

$$v_t = \theta_t + \lambda\,(\widetilde{\theta}_t - \alpha)\,\theta_t$$

Assuming that the alliance is kept into period $t+1$, we have

$$\begin{aligned}
\mathbb{E}_t(v_{t+1}-v_t) = & \int \big((1-\rho)\,\overline{\theta} + \rho\,\theta_t + \epsilon_{t+1}\big)\; dG(\epsilon_{t+1}) + \\
& + \iint \lambda\Big((1-\rho)\,\overline{\theta} + \rho\,\widetilde{\theta}_t + \widetilde{\epsilon}_{t+1} - \alpha\Big)\,\big((1-\rho)\,\overline{\theta} + \rho\,\theta_t + \epsilon_{t+1}\big)\,dG(\epsilon_{t+1})\,dG(\widetilde{\epsilon}_{t+1}) \\
& - \Big(\theta_t + \lambda\,(\widetilde{\theta}_t - \alpha)\,\theta_t\Big) \\
= & \big((1-\rho)\,\overline{\theta} + \rho\,\theta_t\big) + \\
& + \lambda\Big((1-\rho)\,\overline{\theta} + \rho\,\widetilde{\theta}_t - \alpha\Big)\,\big((1-\rho)\,\overline{\theta} + \rho\,\theta_t\big) \\
& - \Big(\theta_t + \lambda\,(\widetilde{\theta}_t - \alpha)\,\theta_t\Big)
\end{aligned}$$

where we use the fact that ϵ_{t+1} and $\widetilde{\epsilon}_{t+1}$ are independent. Since equilibrium assortative matching takes place at time t, we have $\theta_t = \widetilde{\theta}_t > \alpha$, and so

$$\mathbb{E}_t(v_{t+1} - v_t) = -(1-\rho)\,(\theta_t - \overline{\theta}) - \lambda\,(1-\rho)\,\big(\overline{\theta} + (\theta_t - \alpha) + \rho\,(\theta_t - \overline{\theta})\big)\;(\theta_t - \overline{\theta})$$

Since $\theta_t > \alpha$ (by equilibrium condition) and $\theta_t > \overline{\theta}$ (by assumption), the second term is negative and proportional to λ, whence the result follows. ■

This is the stochastic dynamic equivalent of the old adage, "what goes up must come down": firms that form an alliance undergo an increase in value — assuming their type is above average — in a disproportionate way, that is, taking advantage of the complementarity of an alliance between two high-type firms. The counterpart is that, when their type falls, the complementarity effect also implies a disproportionate fall in value. Intuitively, a firm that is matched with a high-θ counterpart under assortative matching is likely to suffer from two negative shocks: a decrease in its own value (own reversion to the mean) and a decrease in the partner's value (partner's reversion to the mean). The second effect (partner's reversion to the mean) is added on to what the firm would be subjected to were it not in an alliance. To make matters worse, the partner's reversion to the mean is particularly significant insofar as the the firm matching a high-θ parter is a high-θ firm itself.

To give a specific example, suppose that the source of complementarity in an alliance comes from bringing together a firm with a high-quality product and a firm with superior market access. The success of such an alliance is roughly proportional to the product of sales margin — which in turn is increasing in quality — and sales quantity. A firm that has a high quality product benefits disproportionately from matching with a firm that can sell such product to many customers. However, such firm is also particularly sensitive to negative shocks to its partner: a given drop in quantity sold is particularly harmful if the firm has a high margin to begin with. Conversely, a firm that sells to a big market benefits from matching up with a firm with a high-quality product. However, such firm is particularly sensitive to negative shocks to its partner's type: a given drop in product quality is particularly harmful if the firm sells to many customers to begin with.

The issue of regression to the mean is also related to the issue of relational rents discussed above. As Proposition 5 shows, the positive effect of an alliance on relational rents is relatively greater for firms with a greater resource level to begin with. However, the relational rents of a higher resource firm are also more likely to drop than those of a lower resource firm.

Wiggins and Ruefli (2005) find evidence that, in the era of hypercompetition, "competitive advantage has become significantly harder to sustain." Interestingly, our analysis suggests that the increase in alliance activity — a trend that would seem contrary to the idea of hypercompetition — also implies that supra-competitive rents tend to vanish more rapidly. However, our results also imply that the size of supra-competitive rents increases as a result of alliances.

A related issue that our dynamic extension allows us to analyze is firm-value volatility, which we define as the time variance of firm value. Proposition 8 suggests that volatility is higher under a regime of firm alliances. The next proposition formalizes this intuition:

Proposition 9. *Suppose that each firm's resource level, θ, evolves according to (3). Contrast two different scenarios: (a) no alliances ever take place; (b) all firms form alliances at $t = 0$ (with assortative matching) and remain allied in future periods. Steady-state volatility of firm value is greater under scenario (b) than under scenario (a).*

Proof: Under scenario (a), from (3) we get

$$\mathbb{V}(\theta) = \rho^2\,\mathbb{V}(\theta) + \mathbb{V}(\epsilon)$$

It follows that the variance of firm value is given by

$$\mathbb{V}(\theta) = \frac{\mathbb{V}(\epsilon)}{1-\rho^2}$$

Under scenario (b), (1) implies that

$$\mathbb{V}(v_{_A}) = \mathbb{V}\Big(\big(1 + \lambda\,(\widetilde{\theta} - \alpha)\big)\,\theta\Big) = \lambda^2\,\mathbb{V}(\theta) + \mathbb{V}(\theta)$$

which is greater than firm-value variance under no alliances. ■

Notice that Proposition 9 applies to *steady-state* levels of volatility and does not depend on the nature of firm matching (assortative or random). The assumption that firm alliances remain in effect indefinitely is rather extreme (and the reason why Proposition 9 applies regardless of the nature of matching). Nevertheless, we believe Proposition 9 points toward an important feature of firm alliances, one to which we return in Section 4.

3. Numerical simulations

In this section, we perform a series of numerical simulations of our model of alliance formation. The purpose is two-fold. First, to illustrate some of the theoretical results from the previous section. And second, to examine the relative role played by positive sorting, selection and information frictions.

■ **The role of assortative matching.** Our first set of simulations examines the role played by positive sorting on the distribution of firm value. As such, it illustrates some of the theoretical results in the previous section.

We generate a series of values of θ, the initial, pre-alliance level of firm value. Specifically, we assume θ is normally distributed with parameters $\mu = 5$ and $\sigma = 1$.[22] We then consider two possible alliance scenarios: random matching (Scenario 1) and assortative matching (Scenario 2). In both cases, we assume that firms have perfect information regarding other firms' resource levels; and that alliance decisions are rational (that is, a firm accepts forming an alliance only when the partner's $\widetilde{\theta}$ is greater than α).

We generate sets of 100 observations each time (that is, we consider an industry with 100 firms). We repeat this process 1000 times, thus generating a total of 100,000 firm-level observations. Regarding the value of α, we assume that $\alpha = \overline{\theta} = 5$. As shown in Proposition 2, this is an important reference value as it implies that randomly matched alliances lead to no change in average firm value. Finally, we assume $\lambda = .03$

Figure 2 depicts the three corresponding densities of the firm value distribution. The dashed line corresponds to pre-alliance value. This is simply the normal distribution that we assume for the value of θ. The two solid lines correspond to post-alliance value distributions. The difference lies in the nature of matching: the light-colored line corresponds to random matching, whereas the dark-colored line corresponds to assortative matching.

First notice that, if firm value is lower than α, then no alliance takes place. This follows from our assumptions of rationality and perfect information. As a result, firm value when $\theta < \alpha$ is the same before and after the alliance price takes place.

Regarding post-alliance value, we see that in both cases (random and assortative matching) there is a rightward shift in distribution (reflecting an increase in firm value). Specifically, the right tail of the value distribution is thicker than the initial distribution of firm value. This is not surprising: by assumption, firms have perfect information regarding each other's resource levels; and alliances are only formed if they increase firm value. The interesting feature depicted in Figure 2 is that, with respect to the initial distribution, the effects of assortative-matching alliances are considerably more significant than the effects of random-matching alliances. Specifically, the right tail of the value distribution is considerably thicker when matching is assortative. This reflects one of the key points in the paper: supermodular matching functions, together with assortative matching, lead the best to form alliances with the best, thus creating "super-best" firms.

22. The normality assumption violates our assumption that $\theta > 0$. However, with $\mu = 5$ and $\sigma = 1$ the probability that $\theta < 0$ is very small, so that the truncated normal (a distributional assumption that is consistent with our assumption) is very close to the normal.

Figure 2
Histogram of firm value before (dashed line) and after (solid lines) alliances take place, assuming perfect information and random matching (Scenario 1, light-colored line) or assortative matching (Scenario 2, dark-colored line)

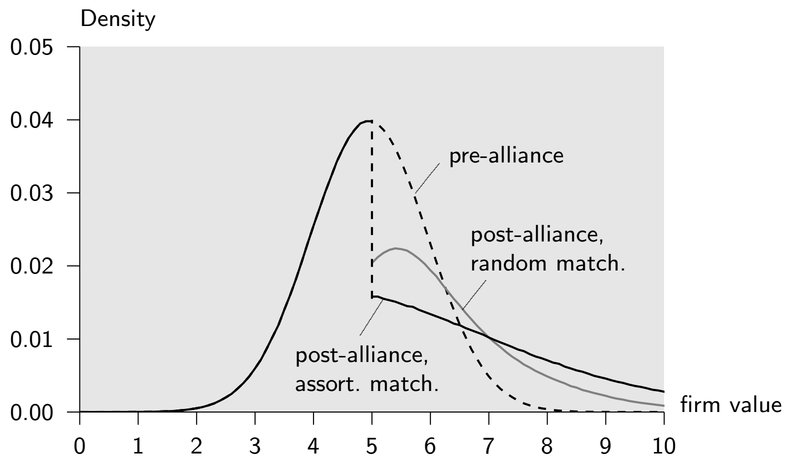


Figure 3
Histogram of firm value before (dashed line) and after (solid lines) alliances take place, assuming assortative matching and perfect information (Scenario 2, light-colored line) or imperfect information (Scenario 3, dark-colored line)

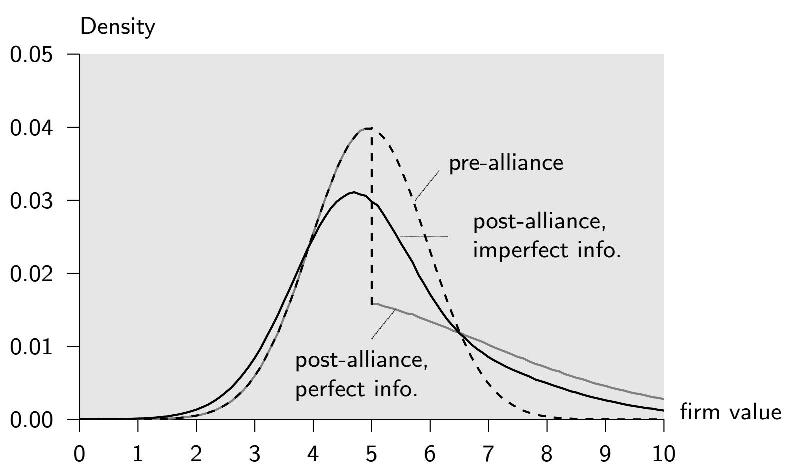
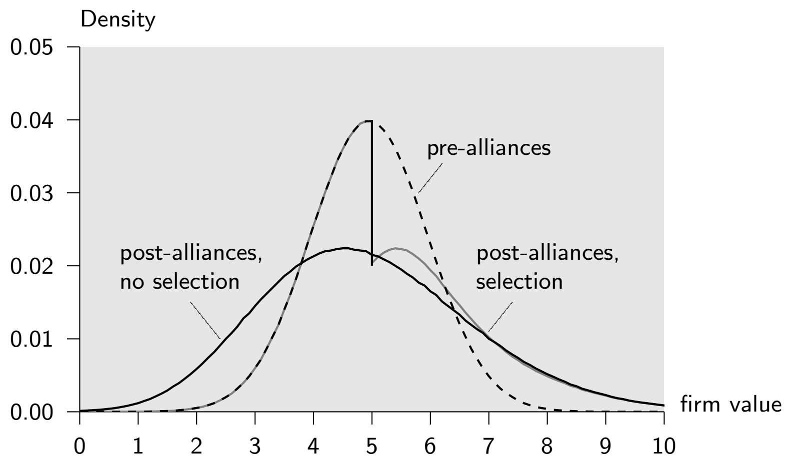


Figure 4
Histogram of firm value before (dashed line) and after (solid line) alliances take place, assuming that all firms are randomly matched and form an alliance



■ **Information frictions.** Our second set of simulations illustrates the effect of imperfect information regarding firm resource levels. Let θ' be the signal observed regarding firm θ's resource level, where $\theta' = \theta + \epsilon$; and where ϵ is normally distributed with zero mean and standard deviation $\sigma_\epsilon = 2$. As shown in the previous section, the natural extension of the stable-matching solution is stable matching in expected value.

As in Figure 2, Figure 3 plots the distribution of initial firm value with a dashed line. The dark lines correspond to the distribution of post-alliance firm value assuming assortative matching. The difference between the two solid lines is that, in one case (light-colored line), we consider assortative matching with perfect information (Scenario 2, already considered in Figure 2), whereas in the other case (Scenario 3, dark-colored line) we consider assortative matching with imperfect information.

In comparison to the dashed line, the dark-colored solid line features an increase in density for values of θ lower than α, that is, we find alliances that reduce firm value. This increase in density at values lower than α results from imperfect information. Although firms are "rational" in rejecting alliances when $\theta' < \alpha$, sometimes they form an alliance with a firm with $\widetilde{\theta} < \alpha$ even though $\widetilde{\theta}' > \alpha$. In other words, imperfect information allows for the possibility of "alliance mistakes" due to measurement error.

Another interesting difference between the the two solid lines is that one of them is discontinuous at $\theta = \alpha$. This reflects our assumption of perfect information, which is obviously somewhat unrealistic (that is, extreme). Given this assumption, all firms with $\theta > \alpha$ form an alliance and no firm with $\theta < \alpha$ forms an alliance. By contrast, assuming that firms observe noisy signals $\widetilde{\theta}'$ of their partner's $\widetilde{\theta}$ leads to a post-alliance density value which is continuous at $\theta = \alpha$.

■ **The role of selection.** As we discussed in the Section 1, value-decreasing alliances can be explained by imperfect information — as we just considered — or by non-optimizing firm behavior, which we now consider. Specifically, we look at the extreme case when *all* firms turn a match into an alliance, regardless of the partner's $\widetilde{\theta}$ value. So as to focus on the role played by the decision to accept a merger proposal, we assume random matching (that is, we ignore the effect of positive sorting). Effectively, this corresponds to assortative matching when $\underline{\theta} \geq \alpha$ and $\sigma_\epsilon/\sigma_\theta \to \infty$. In fact, to the extent that $\sigma_\epsilon/\sigma_\theta \to \infty$ (extreme imperfect information), assortative matching with respect to $\widetilde{\theta}'$ is equivalent to random matching (that is, $\widetilde{\theta}'$ provides no information regarding $\widetilde{\theta}$). In other words, in the limit poor information and poor managerial decisions have similar effects regarding alliance formation.

Figure 4 plots the initial distribution of firm value (dashed line) as well as the distributions

Table 1
Summary statistics of firm value distribution under different scenarios

Secnario	Mean	St.Dev.	Skewness
Pre-alliances	5.00	1.00	1.00
Random matching without selection	5.00	2.08	1.63
Random matching with selection	5.35	2.57	1.87
Assortative matching with imperfect information	5.30	3.05	2.10
Assortative matching with perfect information	5.75	3.70	2.39

resulting from random matching (solid lines). The difference between the solid lines is that one assumes that only when $\theta, \widetilde{\theta} > \alpha$ does a match turn into an alliance (Scenario 1, light-colored line); whereas the other one assumes that all firms turn a match into an alliance (Scenario 4, dark-colored line).

Consistent with Property 2 and our assumption (in this particular simulation) that $\alpha = \overline{\theta}$, under Scenario 4 on average alliances keep firm value at its original level. There is, however, an increase in variance and skewness of the distribution of firm value: supermodular alliance value functions together with random matching imply that there are some very good matches but also some very bad ones. Intuitively, the process of alliance formation works like a "lottery" that adds noise to firm value (sometimes positive, sometimes negative).

Although the process of matching is highly inefficient (random), in a number of cases high-θ firms do get matched with high-$\widetilde{\theta}$ firms, resulting in a very high-value alliance. In Figure 4, this is reflected in a solid-line distributions with a very thick right tail (this is true for random matching with and without selection, that is, under Scenario 1 as well as under Scenario 4).

■ **Summary.** In addition to a distribution of initial values, we considered various possible alliance formation scenarios:

1. Random matching, perfect information.
2. Assortative matching, perfect information.
3. Assortative matching, imperfect information.
4. Random matching, no selection (i.e., all matches are turned into an alliance).

Table 1 presents the summary statistics (mean, standard deviation, skewness) of our various simulations. Specifically, for skewness we use

$$\nu \equiv \sqrt[3]{\int \left(\frac{x - \mu_x}{\sigma_x} \right)^3 f(x)\, dx}$$

where μ_x and σ_x are the mean and standard deviation of the variable in question.

The different scenarios are presented in order of dispersion. Although in theory the ranking according to σ and ν can differ, in our particular case they imply the same ordering. Specifically, the results suggest that firm alliances lead to an increase in the dispersion of firm value, as measured by the second or third centered moments.

The increase in dispersion following firm alliances is seen to depend on several factors. First — and this is one of the central points in our paper — assortative matching is a force in the direction of higher dispersion. The two scenarios featuring positive sorting imply higher σ and μ than the two scenarios with random matching.

Second, given positive sorting, dispersion is greater under perfect information than under imperfect information. Intuitively, better information allows for a better match among equals, which, given a supermodular alliance payoff function, turns into higher dispersion.

Finally, while selection is shown in our simulations to have a *positive* effect on dispersion, we note that, in theory, this is not necessarily the case. As Figure 4 shows, absence of selection implies a thicker left tail (due to "alliance mistakes"). However, under selection a greater weight is placed on alliances between firms with high θ, which, through supermodularity, leads to a thicker right tail. Our simulations suggest that the latter effect dominates the former.[23]

So far we have confined our comments to the effect of alliances on the dispersion of firm value. Table 1 also displays the first moment of the distribution of firm value. First recall that, as anticipated by Property 2, to the extent that $\alpha = \overline{\theta}$ and there is no selection (i.e., every match is turned into an alliance), the average effect of an alliance is zero. However, if there is selection — that is, if firms form an alliance if and only if the alliance leads to an increase in both firms' value — then firm value increases on average — a trivial result. We conclude that selection (i.e., optimal behavior) is a factor contributing to a positive association between alliances and firm value.

Second, Table 1 shows that average post-alliance firm value is greater under assortative matching than under random matching. This results from the supermodular nature of the matching payoff function. As mentioned earlier, positive sorting is a very robust result with respect to the equilibrium concept. In particular, positive sorting is the solution that maximizes total payoff for all participants in the matching game.

Third, given assortative matching, Table 1 shows that average post-alliance firm value is lower under imperfect information. Intuitively, imperfect information creates a friction, which in turn leads to less-than-perfect assortative matching. And, as mentioned in the previous paragraph, perfect positive sorting maximizes total payoff, thus maximizes average payoff as well.

■ **Robustness.** We performed a series of other numerical simulations by changing the values of $\alpha, \mu, \sigma, \sigma_\epsilon$, and λ. The qualitative results presented in the previous paragraphs remain valid, though naturally the actual shape of the density functions is different.

4. Discussion and conclusion

This paper contributes to the "alliance paradox" debate (Kale and Singh, 2009: 45) by examining the effect of alliance formation on the industry distribution of firm value. Alliance formation is considered "one of the most influential" success factors in alliances (Gulati, 1998; Shah and Swaminathan, 2008: 471). Our analytical model subsumes the key elements of alliance formation in the literature, namely (a) partner selection, (b) resource complementarities, and (c) partner commitment and compatibility (Dyer and Singh, 1998; Gundlach et al., 1995; Kale and Singh, 2009; Shah and Swaminathan, 2008). We study two types of alliance formation: a frictionless regime and a regime with informational frictions (e.g., a forecasting error of partner quality). Frictions in alliance formation are examined using numerical simulations.

Our results show that, with or without frictions, alliances increase the dispersion of the firm value distribution. With inter-firm complementarities, better firms benefit disproportionately more from alliances and relational rents become a convex function of partners' resources. Frictionless alliance formation — assortative firm matching and perfect information about partners' resource levels — strengthens our results. These patterns are further amplified when complementarities are endogenously determined by firms' investments. Our paper also shows that, while frictionless alliance formation increases differences in firm relative performance, it does not change firms' relative industry ranking. In addition, entering alliances with informational frictions may lead to value destruction.[24] This phenomenon is particularly salient when industries experience widespread waves of alliances. We also find that the value of an above-average firm regresses to the industry mean faster in an alliance than for standalone firms. The model developed in the paper also proved to

23. We are unaware of any theoretical result to that effect.

24. Aside from information frictions, value-destroying alliances may result from managerial errors or lack of commitment on the part of alliance partners. In terms of our model, all of these situations are captured by the inequality $\widetilde{\theta} \geq \alpha$.

be a useful testbed to audit the relationship between key constructs in prior verbal theories on alliances. We show that complementarities are necessary but not sufficient for alliances to boost firm value, create synergies or produce relational rents — sufficiency also depends on the partner selection regime and the level of resources committed ex-post to the alliance.

Our main theoretical propositions have several empirical implications for firm performance in settings with inter-firm complementarities. Some of these empirical hypotheses are testable using publicly available data. First, industries with higher alliance frequency should exhibit larger dispersion of the firm performance distribution in cross-sectional studies. Second, higher alliance frequency should also be associated with more industry volatility (i.e., variation in firm performance over time). Third, to the extent that the value of firms' resources correlates with business cycles, alliances should amplify the effect of economic downturns/upturns on performance, that is, increase industries' systemic risk. Specifically, improving market conditions boost firms' resource levels, which increases the value of firms in an alliance more standalone firms. Analogously, firm performance should decline more significantly in economic downturns when firms are in alliances. Third, higher alliance frequency in an industry should increase differences in firms' relative performance but also accelerate the regression-to-the-mean of above-average returns. Thus, empirically, we should observe a positive correlation between the magnitude and the transience of firm competitive advantage across industries, ceteris paribus.

The model results also connect to other literatures in strategy. It is plausible that alliances may exacerbate the well-documented entry timing endogeneity problem in studies of first-mover advantages (Hawk et al., 2013; Lieberman and Montgomery, 1988). For example, in international business, if the best multinational firms enter new markets first, they will likely 'take their pick' and partner with the best local companies to overcome the liability of foreignness (Hymer, 1976; Zaheer, 1995). This pattern favors assortative matching and should result in larger differences in firm relative performance. Thus, a positive correlation between firm performance and early entry may be a spurious result of 'double-layered' endogeneity: it is not that early entry leads to higher profits, but that better multinationals enter earlier (the standard endogeneity problem) — and, moreover, that they partner with the best local firms (the alliance endogeneity problem). Empirically, higher frequency of new market entry by alliance should be associated with stronger observed first-mover advantages (partly spurious first-mover advantages). Another empirical setting conducive to a similar phenomenon is the pervasive technological convergence between industries, in which "there is also a growing need for interindustry partnerships and alliances to solve big problems" (E&Y, 2016).

A number of managerial implications also grow out of our analysis. While executives are well aware of ex-post agency and organizational problems in alliances, alliance formation practices should also be closely managed by corporate strategy departments. Synergies, relational rents and value creation must not be too promptly assumed — and the magnitude of inter-firm complementarities, partner selection, and partner resource commitment should be particularly scrutinized. Further, executives should not "over-glorify" alliances as a means to climb up industry rankings. Indeed, in industries with high alliance frequency and limited matching and informational frictions, alliances favor the status quo. Firms must also realize that alliances have a bearing on their strategic choice of (a) variance and (b) volatility. With imperfect information about partners' quality, entering an alliance increases ex-ante variance in firm performance: in good states-of-the-world, the partner is high-quality and complementarities increase firm value; in bad states of the world, the partner is of low quality and alliances destroy value. In some empirical settings (e.g., winners-take-all markets) increasing variance through alliances may be optimal if a firm is underperforming, but suboptimal otherwise (Cabral, 2003). Alliance choices may also be used to calibrate firms' systemic risk. In highly cyclical industries, firms may decide to reduce performance volatility over time by remaining standalone firms. Finally, in international business settings, lower-quality multinationals should be willing to bear the risk of entering new markets earlier to lock in the best local partners — thereby disrupting any process of assortative matching that could worsen their relative performance disadvantage.

Appendix: additional results and proofs

In this appendix we present two additional results referred to in the main text as well as their proofs.

Proposition 10. *Suppose firms are assortatively matched. The skewness of the distribution of firm value increases after alliances have taken place.*

Proof of Proposition 10: van Zwet (1964) proves the following result: if $y = \phi(x)$ is convex and strictly increasing, then all odd standardized central moments of y are higher than the corresponding moments of x. By assumption, a θ type firm gets matched to another θ type firm. An alliance takes place only if $v_{A}(\theta,\theta) > v_{S}(\theta)$, which is true if and only if $\theta > \alpha$. We thus have

$$v_1(\theta) = \begin{cases} \theta & \text{if} \quad \theta < \alpha \\ \theta + \lambda\,(\theta - \alpha)\,\theta & \text{if} \quad \theta > \alpha \end{cases}$$

whereas pre-alliance merger is simply $v_0(\theta) = \theta$. It follows that $v_1(\theta)$ is an increasing convex function of $v_0(\theta)$, and thus the distribution of $v_1(\theta)$ is more skewed than the distribution of $v_0(\theta)$. ■

Proposition 11. *Suppose that all firms are assortitavely matched and form an alliance. If θ is normally distributed with parameters $N(\mu,\sigma)$ and if*

$$\lambda > \frac{2\,\sigma\,(\alpha-\mu)}{2\,\sigma^2 + (\alpha-\mu)^2}$$

then the variance of firm value increases after alliances have taken place. Moreover, the increase in variance is increasing in λ.

Proof of Proposition 11: Suppose θ is normally distributed with mean μ and variance σ. For the purpose of measuring variance, we can change units and reference point so that θ is a standardized normal. The firm value function is then given by

$$v_{A}(\theta,\widetilde{\theta}) = \theta + \lambda(\widetilde{\theta} - \alpha_\circ)\,\theta$$

where $\alpha_\circ = (\alpha-\mu)/\sigma$. Under assortative matching, we have $\widetilde{\theta} = \theta$, and so

$$v_{A}(\theta,\theta) = (1 - \lambda\,\alpha_\circ)\,\theta + \lambda\,\theta^2 \tag{4}$$

If θ is a standardized normal, then

a) $Var(\theta) = 1$
b) $Var(\theta^2) = 2$
c) $Cov(\theta,\theta^2) = 0$

Fact (a) follows from the definition of standardized normal. Fact (b) can be proven as follows: if θ is standardized normal, then θ^2 is chi squared with 1 degree of freedom; and the variance of a chi square with k degrees of freedom is given by $2\,k$. Fact (c) follows from $Cov(\theta,\theta^2) = E(\theta^3) - E(\theta)\,E(\theta^2) = E(\theta^3) = 0$, where the last equality follows from the fact that θ^3, like θ, is symmetric about 0.

As well, recall that the following variance rules apply generally:

$$\begin{aligned} Var(x+y) &= Var(x) + Var(y) + 2\,Cov(x,y) \\ Var(\alpha\,x) &= \alpha^2\;Var(x) \end{aligned}$$

It follows from the preceding facts and from (4) that

$$Var(v_{A}) = (1 - \lambda\,\alpha_\circ)^2 \times 1 + \lambda^2 \times 2 \tag{5}$$

whereas $Var(v_S) = 1$. It follows that $Var(v_A) > Var(v_S)$ if and only if

$$1 - 2\,\lambda\,\alpha_\circ + \lambda^2\,\alpha_\circ^2 + 2\,\lambda^2 > 1$$

or simply

$$\lambda > \frac{2\,\alpha_\circ}{2 + \alpha_\circ^2} = \frac{2\,\sigma\,(\alpha - \mu)}{2\,\sigma^2 + (\alpha - \mu)^2} \tag{6}$$

Moreover, re-writing (5) we get

$$Var(v_A) = 1 + \frac{\lambda}{2 + \alpha_\circ^2}\left(\lambda - \frac{2\,\alpha_\circ}{2 + \alpha_\circ^2}\right)$$

which is increasing in λ whenever (6) holds. ■

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