Supplemental Material

Proof of Proposition 1: Let $p^* = p^*(\lambda, \delta, \psi)$ be the value of p that solves the free entry condition (1). By definition, $\pi^*(\theta', p) = 0$. This implies that the partial derivative of the left-hand side of (1) with respect to p is positive (since profits are increasing in market price). Moreover, straightforward derivation shows that the left-hand side of (1) is increasing in δ and decreasing in λ, whereas the right-hand side is increasing in ψ. Together, this implies that p^* is increasing in ψ and λ, and decreasing in δ.

Straightforward derivation of (2) shows that $\partial r / \partial \lambda > 0$ and $\partial r / \partial x > 0$. Moreover, $\partial r / \partial x \to 0$ as $\lambda \to 0$. Since $\pi^*(\theta, p)$ is increasing in p, both θ' and $x(p)$ are decreasing in p.

Now consider an increase in ψ. From (1) and the discussion above, we conclude this results in an increase in p, which in turn results in a decrease in x, which in turn results in a decrease in r.

Consider now an increase in λ. In this case, I must consider both the direct effect of a change in λ as well as the effect through changes in x. The direct effect on r, given by $\partial r / \partial \lambda$, is positive. However, from (1), an increase in λ leads to an increase in p, which in turn leads to an decrease in x, which in turn leads to an decrease in r, an effect of opposite sign to the previous one. I next show that when $\lambda = 0$ or $\lambda = 1$ then the indirect effect is zero, resulting in a total effect that is positive. Specifically,

$$\frac{dr}{d\lambda} = \frac{\partial r}{\partial \lambda} + \frac{\partial r}{\partial x}\,\frac{dx}{d\lambda} = \frac{1 - x}{\big(1 - x\,(1 - \lambda)\big)^2} + \frac{\lambda\,(1 - \lambda)}{\big(1 - x\,(1 - \lambda)\big)^2}\,\frac{dx}{d\lambda}$$

The first term is positive for all values of λ. Since $dx / d\lambda$ is finite, the second term is zero for $\lambda = 0$ or $\lambda = 1$. ■

Proof of Proposition 2: Define

$$E^+(z) \equiv E(\theta \mid \theta > z) = \frac{\int_z^\infty \theta\, dF(\theta)}{\int_z^\infty dF(\theta)}$$

Straightforward derivation reveals that $E^+(z)$ is increasing. Average productivity is then given by

$$\bar{\theta} = \frac{n}{m}\, E^+(0) + \frac{m - n}{m}\, E^+(\theta')$$

It follows that

$$\frac{d\bar{\theta}}{d\psi} = \left(\frac{m - n}{m}\right)\left(\frac{dE^+(\theta')}{d\psi}\right) + \left(\frac{d\,\frac{n}{m}}{d\psi}\right)\Big(E^+(\theta') - E^+(0)\Big)$$

By the same argument as in the proof of Proposition 1, (2) implies that, if $\lambda = 0$ or $\lambda = 1$, then $dr / d\psi = 0$. This and the fact that r is continuously differentiable in λ implies that, for λ close to 0 or close to 1

$$\frac{d\bar{\theta}}{d\psi} \approx \frac{m - n}{m}\left(\frac{dE^+(\theta')}{d\psi}\right) \tag{3}$$

As shown in the proof of Proposition 1, p is increasing in ψ and θ' is decreasing in p. It follows that θ' is decreasing in ψ, and so the right-hand side of (3) is negative.

A similar argument holds for variation in λ. ■