Review of Economic Dynamics 20 (2016) 43–62

Contents lists available at ScienceDirect

Review of Economic Dynamics

www.elsevier.com/locate/red

Dynamic pricing in customer markets with switching costs

Luís Cabral [1,2]

New York University and CEPR, United States

A R T I C L E I N F O

Article history:
Received 20 March 2013
Received in revised form 4 September 2015
Available online 18 September 2015

JEL classification:
L1
L4

Keywords:
Switching costs
Dynamic pricing
Customer markets

A B S T R A C T

In a dynamic competitive environment, switching costs have two effects. First, they increase the market power of a seller with locked-in customers. Second, they increase competition for new customers. I provide conditions under which switching costs decrease or increase equilibrium prices. Taken together, the results suggest that, if markets are very competitive to begin with, then switching costs make them even more competitive; whereas if markets are not very competitive to begin with, then switching costs make them even less competitive. In the above statements, by "competitive" I mean a market that is close to a symmetric duopoly or one where sales take place with high frequency.

1. Introduction

Buyers (firms or final consumers) frequently must pay a cost in order to switch from their current supplier to a different supplier. These costs suggest some interesting questions: are markets more or less competitive in the presence of switching costs? Are prices higher or lower under switching costs? How do seller profits and consumer surplus vary as switching costs increase?

Empirical evidence regarding these questions is ambiguous: although most studies suggest that switching costs lead to higher prices, there is also evidence to the contrary. This ambiguity is mirrored by the theoretical literature: some papers provide sufficient conditions such that switching costs make markets less competitive, while others predict the opposite effect.

In this paper, I develop an analytical framework that, while fairly parsimonious, is flexible enough (and largely functional form independent) to encompass several possibilities, including pro- and anti-competitive effects of switching costs. In developing and analyzing this framework, I hope to provide intuition for some of the main effects of switching costs on market competition and reconcile apparently divergent results in the literature.

Although fairly general in several dimensions, I restrict my analysis to the case when sellers can discriminate between locked-in and not locked-in consumers.[3] This is a reasonable approximation for the workings of many intermediate goods

E-mail address: luis.cabral@nyu.edu.

[1] Paganelli-Bull Professor of Economics and International Business, Stern School of Business, New York University; Research Fellow, IME and PPRS (IESE); and Research Fellow, CEPR.

[2] I am grateful to the Associate Editor Matt Mitchell and to two referees for very helpful comments and corrections. The usual disclaimer applies.

[3] Throughout the paper I will use both the terms "locked-in" and "attached."

http://dx.doi.org/10.1016/j.red.2015.09.002
1094-2025/

markets — from aircraft to ready-mixed concrete — where sales contracts are typically tailored to each customer and their purchase history is known. These are frequently referred to as *customer markets* (thus the paper's title).[4]

Focusing the analysis on customer markets has two advantages. First, much of the previous research has centered on markets with many buyers and sellers who cannot discriminate between buyers, thus excluding an important class of examples (especially in intermediate goods markets). Second, the analysis of competition for a single buyer allows me to consider a fairly general dynamic framework, thus avoiding many of the limitations of the previous research on the topic.

Most of the prior economics literature, especially the early literature, solved some variation of a simple two-period model (see Section 2.3.1 in Farrell and Klemperer, 2007). The equilibrium of this game typically involves a bargain-then-ripoff pattern: in the second period, the seller takes advantage of a locked-in consumer and sets a high price (ripoff). Anticipating this second-period profit, and having to compete against rival sellers, the first-period price is correspondingly lowered (bargain). One limitation of two-period models is that potentially they distort the relative importance of bargains and ripoffs. In particular, considering the nature of many practical applications, two-period models unrealistically create game-beginning and game-ending effects. To address this problem, I consider an infinite-period model where the state variable indicates the seller to which a given consumer is currently attached.

The core section of the paper, Section 3, deals with the central research question in the switching costs literature: whether switching costs decrease or increase market competitiveness. In a two-period model, the answer is: lower-price bargains in the first period and higher-price ripoffs in the second period. The dynamic counterpart to the bargain-then-ripoff pattern is given by two corresponding effects on a seller's dynamic pricing incentives: the harvesting effect (sellers with locked-in customers are able to price higher without losing much demand) and the investment effect (sellers without locked-in customers are eager to cut prices in order to attract new customers).

The harvesting and investment effects work in opposite directions in terms of market average price. Which effect dominates? Conventional wisdom and the received economics literature suggest that the harvesting effect dominates (Farrell and Klemperer, 2007). However, as I mentioned earlier, recent research casts doubt on this assertion.[5] In Section 3, I attempt to clarify the issue by providing conditions under which switching costs decrease or increase equilibrium prices. The bottom line is that, if markets are very competitive to begin with, then switching costs make them even more competitive; whereas if markets are not very competitive to begin with, then switching costs make them even less competitive. In the above statements, by "competitive" I mean a market where each firm has approximately the same probability of attracting each given customer; or one where the discount factor is very high, so that the competition for future customers is relatively more important than revenues from current customers. In other words, in very competitive markets the investment effect dominates, whereas in non-competitive markets the harvesting effect dominates.

In Section 4, I consider a series of additional results regarding entry barriers, profits and welfare, and customer recognition. In Section 5, I discuss various possible extensions. Along the way, I try to relate my framework to the previous literature, thus organizing the literature's main themes around a single analytical framework. (For this reason, I omit a systematic literature review in the present section.) Finally, Section 6 concludes the paper.

2. Model and preliminary results

Consider an industry where two sellers compete over an infinite number of periods for sales to n infinitely lived buyers. Sellers' discount the future according to discount factor $\delta \in (0,1)$, buyers according to $\beta \in (0,1)$. Each buyer's valuation for firm i's product is given by $\alpha + \xi_i$, where α is a constant and $\xi_j = -\xi_i$, that is, ξ_i is the relative preference for firm i's product.[6] I assume that the buyer's outside option is worth $-\infty$, so the market is "covered" (that is, the buyer always chooses one of the firms).[7] I also assume that ξ_i is iid across periods. (Later in the paper I depart from this assumption, considering the alternative where relative preferences evolve according to an auto-regressive process.)

An important assumption throughout the paper is that sellers are able to discriminate between locked-in and not locked-in buyers (that is, buyers who are locked in to the rival seller). Without further loss of generality, I hereafter focus on the sellers' competition for a particular buyer. (A seller's total market share and total value can then be determined by simple aggregation over all buyers.)

Within each period, the timing is as follows: First nature generates ξ_{it}. Next sellers simultaneously set prices p_{it}. Then the buyer chooses one of the sellers. Finally, period payoffs are received: assuming production costs are zero, seller i receives p_{it} if a sale is made and zero otherwise.[8] The buyer receives $\alpha + \xi_{it} - p_{it} - I_t\, s$, where I_t an indicator variable:

[4] The assumption that sellers can discriminate between locked-in and not-locked-in consumers also applies to final good products such as magazine subscriptions and bank accounts (Klemperer, 1995, suggests many other examples).

[5] The idea that the effects of switching costs can be pro-competitive is not novel. See for example von Weizsacker (1984) and Klemperer (1987a, 1987b) for seminal contributions and Cabral and Villas-Boas (2005) for a reinterpretation of some of those results. Recent research has explored the competitive effects of switching costs in an infinite-period context.

[6] An alternative way of modeling preferences would be to assume the preference for firm i's product is given by ζ_i, distributed according to cdf $\Phi(\zeta_i)$, and then define $\xi_i \equiv \zeta_i - \zeta_j$.

[7] Alternatively, I can assume that the outside option is zero and that α is high enough that one of the products is chosen with probability 1. However, for this alternative interpretation to work, the $s \to \infty$ limit results must implicitly assume a sequence of values $\alpha(s)$.

[8] Alternatively, I could assume that both sellers have the same production cost, in which case prices should be interpreted as margins above cost.

$$I_t = \begin{cases} 0 & \text{if the seller at } t-1 \text{ is the same as the seller at } t \\ 1 & \text{if the seller at } t-1 \text{ is different from the seller at } t \end{cases}$$

Regarding the distribution of ξ_{it}, which has cdf $F(\cdot)$ and density $f(\cdot)$, I make the following assumptions:

Assumption 1. (i) $F(x)$ is continuously differentiable; (ii) $f(x) = f(-x)$; (iii) $f(x) > 0$, $\forall x \in \mathbb{R}$; (iv) $f(x)$ is unimodal; (v) $F(x)/f(x)$ is strictly increasing.

Many distribution functions, including the Normal and the t, satisfy Assumption 1. In many of the results that follow, I will use repeatedly the following lemma, which characterizes several properties of F that follow from Assumption 1:

Lemma 1. *Under Assumption 1, the following are strictly increasing in x:*

$$\frac{F(x)^2}{f(x)}, \qquad \frac{F(x)-1}{f(x)}, \qquad \frac{2F(x)-1}{f(x)}$$

Moreover, the following is increasing in x iff $x > 0$ and constant in x at $x = 0$:

$$\frac{\left(1-F(x)\right)^2 + \left(F(x)\right)^2}{f(x)}$$

(The proof of this and the following results may be found in the Appendix A.)

I focus on symmetric Markov equilibria, where the state of the game is defined by the identity of the firm currently "holding" the consumer in question. I refer to this firm as the "insider" and denote it by a subscript 1, whereas the other firm is referred to as "outsider" and denoted by a subscript 0.

To the extent that F is symmetric, the model itself is symmetric. For this reason, the game looks the same regardless of the identity of the "insider." In other words, while this simplest version of the model allows for two states, these two states are identical up to a permutation of firms. Nevertheless, these are two non-trivial states: firm prices depend on their current status as "insider" or "outsider." (Later in this section I consider the alternative case when F is not symmetric.)

Symmetry and serially uncorrelated preferences imply that the buyer's continuation values from being attached to seller i or seller j are the same. This greatly simplifies the analysis, for even forward looking buyers need not compute any value function. (When I depart from the assumptions of symmetry and serially uncorrelated preferences I will need to compute buyer value functions explicitly.)

In each period the buyer chooses the insider seller if and only if

$$\xi_1 - p_1 \geq -p_0 - s \tag{1}$$

Define

$$x \equiv p_1 - p_0 - s \tag{2}$$

Then (1) may be re-written as $\xi_1 \geq x$. In words, x is the critical level of the buyer's relative preference ξ_1 such that the buyer chooses the insider seller. Denote by q_1 and q_0 the probability that the buyer chooses the insider or the outsider seller, respectively. Since ξ_1 is distributed according to $F(\cdot)$, we have

$$q_1 = 1 - F(x)$$
$$q_0 = F(x)$$

While the buyer's value functions are of no importance at this stage, the sellers' value functions are quite central to the arguments that will follow. I denote the seller's value by v_i, where $i = 0, 1$ stands for outsider and insider seller, respectively.

As mentioned earlier, my analysis focuses on Markov equilibria. My first result shows that there exists only one such equilibrium that is symmetric.

Proposition 1. *There exists a unique symmetric Markov equilibrium.*

Proposition 1 provides the starting point for the main results in the paper, where I perform a series of comparative statics exercises with respect to the value of switching cost s.

3. Results: bargains and ripoff

The central question in the literature on switching costs is the effect that these costs have on market competition: do they lead to higher or to lower prices? As mentioned earlier, the literature is somewhat ambiguous and somewhat confused in this respect: some models show an increase in s leads to higher prices, some models show the reverse effect; and the reason why that's the case is not always clear.

In this section, I show that even a parsimonious model such as the one I consider allows for both possibilities (pro- and anti-competitive switching costs). I also attempt to provide some intuition in a way that reconciles previous results in the literature within a reasonably unified framework.

First I show that switching costs lead to an increase in the insider's price and a decrease in the outsider's price.

Proposition 2. *There exists a $\delta' \in (0,1)$ such that, if $\delta < \delta'$, then $dp_1/ds > 0$ and $dp_0/ds < 0$*

In words, switching costs imply that the insider firm sets a "ripoff" price, whereas the outsider firm sets a "bargain" price. As Farrell and Klemperer (2007) mention, "the bargain-then-ripoff structure is clearest when new and locked-in customers are clearly distinguished and can be charged separate bargain and ripoff prices, respectively" (as is the case in customer markets). This chronological bargain-then-ripoff pattern is typical in first-generation two-period models such as Klemperer (1987a, 1987b). Proposition 2 suggests that a similar ripoff–bargain dichotomy is found in infinite period models.

Chen (1997) considers a two-period model where firms offer discounts to new customers. The so-called strategy of "paying customers to switch" effectively corresponds to Proposition 2's prediction that $p_0 < p_1$ (though the latter corresponds to an infinite period model rather than to a two-period model).[9] Fabra and García (2012) develop a continuous-time dynamic model with switching costs. In their Markov Perfect equilibrium, the dominant firm concedes market share by charging higher prices than the smaller firm, a result that is consistent with Proposition 2. Similar dynamics, though within a different framework, appear in Rhodes (2014).

One important difference of Proposition 2 with respect to earlier (two-period) models is that, more than "bargain-then-ripoff," what we observe is "bargain *and* ripoff:" in each period, consumers choose between being ripped off by their "current" supplier (that is, the supplier they purchased from in the previous period); or getting a bargain from the outside supplier (though paying a switching cost s). To the extent that consumers do not switch in equilibrium (as is the case in most "first generation" models), this effectively translates into a sequence of bargain followed by ripoff. However, in models with private information about consumer preferences such as my model, switching does occur in equilibrium; and so a consumer's life is a sequence of bargains and ripoffs.

There is a second, perhaps more important, difference between the present model and early switching cost models. Modeling competition as a two-period game introduces potential distortions in firm and consumer behavior. Specifically, it introduces an artificial "end-of-world" effect: in period 2, an insider firm has little to lose from ripping off a locked-in consumer. However, such "end-of-world" effects seem more of a modeling trick than a feature of reality. In fact, the qualification $\delta < \delta'$ in Proposition 2 is necessary: if the future is sufficiently important for firms, then there are no ripoffs: switching costs lead to lower prices by both insider and outsider firm. More on this later in this section.

Proposition 2 shows that, if the discount factor is sufficiently low, then switching costs increase the insider's price and decrease the outsider's price. What about average price? In other words, which effect dominates: the insider's price increase or the outsider's price decrease? Specifically, let $\bar{p}$ be the average price paid by the buyer, that is,

$$\bar{p} = \sum q_i \, p_i$$

My next goal is to characterize average price $\bar{p}$ as a function of switching costs s. I first show that if s is small (resp. large) then an increase in s leads to a decrease (resp. increase) in $\bar{p}$.

Proposition 3. *For each value of δ, there exist values s', s'', where $0 < s' < s'' < \infty$, such that: (a) If $s < s'$, then average price $\bar{p}$ is decreasing in switching cost s; (b) If $s > s''$, then average price $\bar{p}$ is increasing in switching cost s.*

To the best of my knowledge, Dubé et al. (2009) were the first to show, by means of numerical computations, that small switching costs may lead to lower prices.[10]

To understand the intuition for Proposition 3, it is useful to look at the sellers' first-order conditions. The insider seller's value function is given by

$$v_1 = \big(1 - F(x)\big)(p_1 + \delta\, v_1) + F(x)\, \delta\, v_0 \tag{3}$$

where v_i is seller i's value. In words: with probability $1 - F(x)$, the insider seller makes a sale. This yields a short-run profit of p_1 and the continuation value of an insider, v_1. With probability $F(x)$, the insider loses the sale, makes zero short run profits, and earns a continuation value v_0.

[9] See also Shaffer and Zhang (2000).

[10] Cabral (2009) includes an early version of the first part of Proposition 3.

Maximizing with respect to p_1, we get the insider seller's first-order condition:

$$p_1 = \frac{1 - F(x)}{f(x)} - \delta V \tag{4}$$

where $V \equiv v_1 - v_0$ is the difference, in terms of continuation value, between winning and losing the current sale. In other words, $-\delta V$ is the "cost," in terms of discounted continuation value, of winning the current sale.

Since $q_1 = 1 - F(x)$ and $x = p_1 - p_0 - s$, we have $\frac{dq_1}{dp_1} = f(x)$. It follows that (4) may be re-written as

$$\frac{p_1 - (-\delta V)}{p_1} = \frac{1}{\epsilon_1}$$

where $\epsilon_1 \equiv \frac{dq_1}{dp_1} \frac{p_1}{q_1}$. This is simply the "elasticity rule" of optimal pricing, with one difference: the future discounted value from winning the sale appears as a negative cost (or subsidy) on price.

We thus have two forces on optimal price, which might denoted by "harvesting" and "investing" (I borrow the terms from Farrell and Klemperer, 2007). If the seller is myopic ($\delta = 0$), then optimal price is given by the first term in the right-hand side of (4). The greater the value of s, the smaller the value of x (as shown in the proof of Proposition 3), and therefore the greater the value of p_1. We thus have harvesting, that is, a higher switching cost implies a higher price (by the insider seller, which is the more likely seller).

Suppose however that $\delta > 0$. Then we have a second effect, investing, which leads to lower prices. The greater the value of s, the greater the difference between being an insider and being an outsider, that is, the greater the value of V.

What is the relative magnitude of the harvesting and the investment effects on average price? First notice that harvesting leads to a higher price by the insider but a lower price by the outsider. If fact, by symmetry, the effects are approximately of the same absolute value when s is close to zero. To see this, consider the outsider's problem. The value function is given by

$$v_0 = F(x)(p_0 + \delta v_1) + \big(1 - F(x)\big)\delta v_0 \tag{5}$$

The first-order condition for value maximization is then

$$p_0 = \frac{F(x)}{f(x)} - \delta V \tag{6}$$

The "static" component of the first-order condition (corresponding to $\delta = 0$) moves in the opposite direction of p_1, as can be seen from (4). From a short-run point of view, the switching cost s effectively introduces vertical product differentiation between the insider and the outsider, which in turn leads the insider to increase price and the outsider to reduce price by the same amount. This implies that, for s close to zero and in terms of average price, the harvesting and investment effects approximately cancel out, since for $s = 0$ insider and outsider sell with equal probability.[11]

Not so with the dynamic effect. In fact, as can be seen from (4) and (6), the "subsidy" resulting from the value of winning is the same for insider and outsider; and, to the extent that $v_1 > v_0$, the subsidy is positive for both players, that is, both want to price aggressively so as to secure a better position in the future. It follows that the effect on average price is unambiguously negative, and of first-order importance.

In other words, the harvesting effect is symmetric: the amount by which the insider increases its price is the same as the amount by which the outsider lowers its price. However, the investment effect is equal for both sellers — and negative. Specifically, the dynamic effect is proportional to $V \equiv v_1 - v_0$, as mentioned earlier. Substituting (4) for p_1 in (3); and substituting (6) for p_0 in (5); and simplifying, we get

$$v_1 = \frac{\big(1 - F(x)\big)^2}{f(x)} + \delta v_0$$

$$v_0 = \frac{F(x)^2}{f(x)} + \delta v_0$$

$$V = v_1 - v_0 = \frac{\big(1 - F(x)\big)^2}{f(x)} - \frac{F(x)^2}{f(x)} = \frac{1 - F(x)}{f(x)} - \frac{F(x)}{f(x)} \tag{7}$$

If $s = 0$, then $x = 0$ and $V = 0$, that is, there is no value of being an insider firm. However, for $s > 0$ we have $x < 0$ and $V > 0$; and the greater s is, the greater V is: switching costs imply that a customer base is an asset.

Now consider the case when s is very high. This implies a low value of x, which in turn implies a low value of $F(x)$. In other words, a high s implies that the insider's probability of a sale, $1 - F(x)$, is close to 1. This implies that the average

11 Arie and Grieco (2012) rephrase this intuition as follows: switching costs work like a subsidy to a firm's existing customers and a tax to everybody else. Since at $s = 0$ duopolists have exactly half the market (i.e., probability of sale), the tax and subsidy effects cancel.

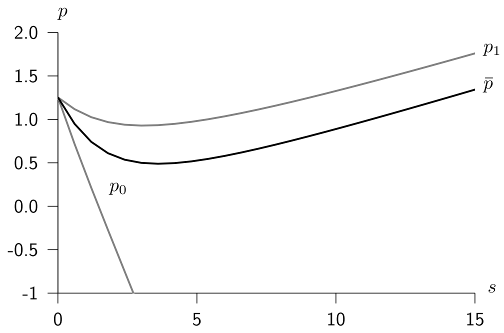

Fig. 1. Switching cost and equilibrium price ($\delta = .9$). Lighter lines represent seller prices; dark line represents average price.

price paid by the buyer is essentially determined by the insider's price. Substituting (31) for V in (4), we derive that the insider's price is given by

$$p_1 = \frac{1 - \delta - (1 - 2\,\delta)\,F(x)}{f(x)} \tag{8}$$

As $s \to \infty$, $x \to -\infty$ and $p_1 \to \infty$. This is the classic "ripoff" effect of switching costs: once the insider locks in the buyer, if valuations and the cost of switching are very high then the insider can set a very high price.

Fig. 1 illustrates Proposition 3. (In all numerical illustrations in this paper, I assume ξ is distributed according to a standardized normal.) On the horizontal axis, the value of switching cost varies from zero to positive values. On the vertical axis, three prices are plotted: the insider's price, the outsider's price, and average price. As the figure shows, average price is a U-shaped function of switching cost: for low values of s, higher switching costs have a pro-competitive effect, whereas for higher values of s, higher switching costs have an anti-competitive effect.

In sum, Proposition 3 suggests that the effect of switching costs on prices is largely an empirical question. Dubé et al. (2009) claim that, for various products, the value of switching cost lies in the region when the net effect of switching costs is to decrease average price.

Earlier dynamic models of price competition with switching costs predicted an anti-competitive effect of an increase in s: Beggs and Klemperer (1992), Farrell and Shapiro (1988), Padilla (1995). In particular, Padilla (1995) shows that "switching costs unambiguously relax price competition." One important difference of these papers with respect to my framework is that they assume sellers cannot discriminate between locked-in and non-locked-in customers.

Shin and Sudhir (2009) argue that the "U-shaped relationship between switching cost and market competitiveness" found in Dubé et al. (2009) depends crucially on "preference heterogeneity (product differentiation) and changing preferences over time."[12] My Proposition 3 shows that their assumptions regarding changing preferences are not necessary in order to obtain the empirically observed U-shaped pattern. By contrast, the assumption of product differentiation plays an important role. In fact, Farrell and Shapiro (1989) show that, absent product differentiation, switching costs have no effect on prices. In fact, assuming sellers can negotiate efficient contracts with buyers, prices exactly adjust for switching costs; and in equilibrium switching does not take place.[13]

For the particular case of linear preferences and linear pricing strategies, Doganoglu (2010) also shows that an increase in s leads to a decrease in $\bar{p}$. Also for linear preferences, Rhodes (2014) provides conditions such that long-run equilibrium prices are lower under switching costs, allowing for buyer preferences to be correlated across time (see also Section 4). Finally, Fabra and García (2012) develop a continuous-time dynamic model with switching costs and conclude that "as market structure becomes more symmetric, price competition turns fiercer and in the long-run, switching costs have a pro-competitive effect." All of these results are consistent with Proposition 3, though they are restricted to specific functional forms; I will return to these papers later in this section.

High discount factor. I have just shown that, for any preference distribution F satisfying Assumption 1 and any positive discount factor δ, average price is a U-shaped function of s. I next provide an alternative sufficient condition for competitive switching costs: for any preference distribution F satisfying Assumption 1 and for any positive value of the switching cost s, if the discount factor is sufficiently close to 1 then average price is decreasing in s.

Proposition 4. *For each value of s, there exists a $\delta' \in (0, 1)$ such that, if $\delta > \delta'$, then average price $\bar{p}$ is decreasing in switching cost s.*

---

[12] These features, heterogeneity in consumer preferences and changing preferences, are also present in Shin and Sudhir (2009), which addresses the issue of whether or not to discriminate between locked-in and non-locked-in consumers.

[13] See also the discussion on product differentiation in Section 5.

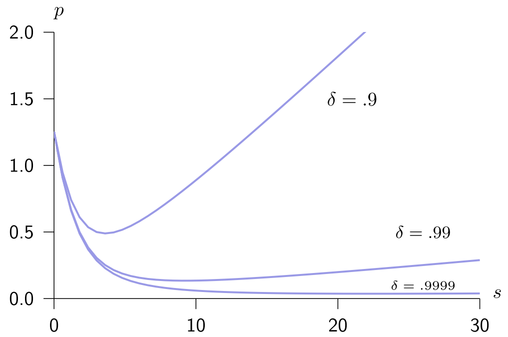

Fig. 2. Switching cost and equilibrium average price as a function of the sellers' discount factor.

Fig. 2 illustrates Proposition 4. It plots average price as a function of s for various values of δ. The $\delta = .9$ curve corresponds to the curve in Fig. 1. It is U shaped: for small values of s, average price is decreasing in s (Proposition 3). However, for high values of s, average price becomes increasing in s. As we consider higher values of δ, the U shape becomes more and more extended, so that, for a given range $[0, \bar{s}]$ of values of s, average price eventually becomes uniformly decreasing in s (Proposition 4).

As in repeated games and other areas, one may interpret changes in the discount factor in different ways. The most direct interpretation is as a change in the rate at which agents discount the future, that is, the continuous time discount rate. However, most of the variation in discount factor across industries is more likely to be due to other factors. Most important among these is the frequency with which decisions are made. This should be the favored interpretation, that is, we should think of the discount factor as given by $\delta = e^{-r\Delta}$, where Δ is the time interval between two consecutive decisions; and changes in δ as resulting from changes in Δ. In this context, Proposition 4 may be re-stated as follows: *the higher the frequency with which consumers make purchases, the more likely an increase in switching costs leads to a decrease in average price.* I return to this issue at the end of this section.

I next attempt to provide an intuitive explanation for Proposition 4. If $\delta = 0$, seller value is given by short-run profit, the first term on the right-hand side of the value functions (7). In a dynamic equilibrium, seller value is given by these short-term profits plus δv_0, *regardless of whether the seller wins or loses the current sale*.

This is an important point and one worth exploring in greater detail. To understand the intuition, it may be useful to think of an auction with two bidders with the same valuations. Specifically, each bidder gets w if he wins the auctions and l if he loses. It follows that the Nash equilibrium is for both bidders to bid $w - l$. If follows that the equilibrium value is l for both bidders (winner or loser). In other words, the extra gain a bidder receives from being the winner, $w - l$, is bid away, so that a bidder can't expect more than l.

In the dynamic game at hand the analog of l is the continuation value if the seller loses the current sale, δv_0. So the idea is that all of the extra gain in terms of future value, $\delta V = \delta (v_1 - v_0)$, is bid away in the form of lower prices. In the limit as $\delta \to 1$, this implies that the insider's position is as good as that of an outsider. In fact, taking limits in (8), we get

$$\lim_{\delta \to 1} p_1 = \frac{F(x)}{f(x)} \tag{9}$$

But, as we can see from (6), the right-hand side of (9) is simply the equilibrium value of p_0 when $\delta = 0$. In words, as the discount factor tends to 1, the insider's price level converges to the outsider's static price level (i.e., when $\delta = 0$). But we know that, in a static model, increasing vertical product differentiation (in particular, increasing the switching cost) leads to a lower price by the "outsider" seller. That is, as we increase s from zero to a positive value, keeping $\delta = 0$, then the insider's price increases and the outsider's price decreases. If $s = 0$, then equilibrium price is the same regardless of the value of δ. Finally, putting it all together, we conclude that, as $\delta \to 1$ and $s > 0$, the high price is at the level of the lower price when $s = 0$; and so switching costs lead to lower average price.

In a model with linear preferences, Villas-Boas (2006) shows that prices are lower if sellers are more forward-looking, a result that is consistent with Proposition 4. Padilla (1992) finds that the presence of new customers due to market expansion always leads to lower prices. A growing market effectively corresponds to a higher discount factor δ, so his result is broadly consistent with Proposition 4. However, differently from my result, he argues that overall prices and profits are still higher than in the absence of switching costs. However, this conclusion relies on the end-of-the-world effect of his two-period model.

Asymmetric duopoly. Proposition 3 provides sufficient conditions for switching costs to be pro-competitive or anti-competitive (low or high s, respectively). Proposition 4 provides an alternative set of sufficient conditions for switching costs to be pro-competitive (high δ). I next provide an alternative set of conditions such that switching costs are anti-competitive: seller asymmetry.

Suppose that the buyer has a fixed preference d for seller A. In order to reduce the number of equations to be written, below I use the notation d_i for the buyer's preference for firm i, where $d_A = d$ and $d_B = -d$. As before, I use the subscripts 0

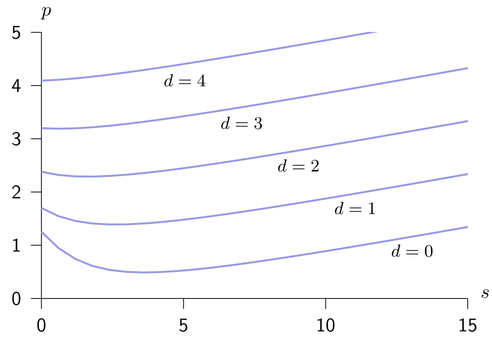

Fig. 3. Switching cost and equilibrium average price in asymmetric duopoly ($\delta = .9$, $\beta = 0$).

and 1 to denote the current state of a firm, outsider or insider, respectively. As before, I have a Markov model with two states: seller A is the insider or seller B is the insider. Differently from before, I now need to account for the insider's identity explicitly.

A buyer currently attached to seller i prefers to stay with that seller if and only if

$$d_i + \xi_i - p_{1i} + \beta\, u_i \geq -s - p_{0j} + \beta\, u_j$$

where β is the buyer's discount factor and u_i the discounted buyer's value from being attached to firm i, measured *before* the buyer learns his valuations for that period. The critical values x_i leading the buyer to purchase from seller i when seller i is the insider are now given by

$$x_i \equiv p_{1i} - p_{0j} - \beta\,(u_i - u_j) - s - d_i \tag{10}$$

As before, seller $1i$ (the insider) sells with probability $q_{1i} = 1 - F(x_i)$.

Notice that, up to now, given my assumption that the outside option is worth $-\infty$ and that firms are symmetric, I did not need to compute consumer utility, using ξ_i as a sufficient statistic for consumer choice. Now I need to explicitly compute the consumer value function, which requires that I compute consumer utility in each period, which in turn requires that I go back to the primitive valuations. Taking this into account, the buyer value functions u_i are recursively given by

$$u_i = \alpha + G(x_i) + G(-x_i) + \big(1 - F(x_i)\big)(d_i - p_{1i} + \beta\, u_i) + F(x_i)(d_j - s - p_{0j} + \beta\, u_j) \tag{11}$$

$i = A, B,\ \ j \neq i$, where

$$G(x) \equiv \mathcal{P}(\xi > x)\; E\big(\xi \mid \xi > x\big) = \int_x^\infty \xi\, dF(\xi)$$

is the buyer's expected valuation (net of the constant α) given that he chooses a particular seller by using the threshold x of differences in valuations (times the probability of choosing that particular seller).

My next result shows that asymmetry matters for the impact of switching costs on average price.

Proposition 5. *There exists a d' such that, if $d > d'$, then: there exist $\beta'(d), \delta'(d) \in (0, 1)$ such that, if $\beta < \beta'(d)$ and $\delta < \delta'(d)$, then an increase in switching costs leads to an increase in average price.*

To understand the intuition for Proposition 5, recall that, if s is not too small and δ is not too high, then an increase in switching costs leads to a decrease in the outsider's price and an increase in the insider's price ("bargain and ripoff" effect). If the industry is relatively symmetric (that is, if d and s are close to zero), then the "bargain" effect counterbalances the "ripoff" effect: in equilibrium, a consumer is almost as likely to go with the insider ("ripoff") or the outsider ("bargain"). However, if one of the firms is clearly preferred by consumers (independently of the switching cost) then most of the time the insider is the firm the consumer has a special preference for; and switching occurs with very low probability. In this context, the "ripoff" effect of switching costs dominates. In other words, if the discount factor is not too large, then for d high enough the "harvesting" effect dominates the "investment" effect.

Fig. 3 illustrates the effect of increasing d. For $d = 0$, we see that average price as a function of switching cost follows the U-shaped pattern described earlier (see Fig. 1). By contrast, as the value of d increases, the U shape turns into a uniformly increasing line: an increase in switching costs leads to a higher average price regardless of the level of s. Also, while Proposition 5 requires that the values of the discount factors be sufficiently small, Fig. 3 suggests that the same result holds for higher value of the discount factor δ.

In a fairly different framework, Fabra and García (2012) also conclude that "if firms are sufficiently asymmetric, an increase in switching costs also leads to higher current prices."[14] I will return to this below.

Serially correlated preferences. Up to now, I assumed that buyer preferences are serially uncorrelated. Suppose instead that a buyer's relative preference for seller i and time t is given by

$$\omega_{it+1} = \rho\, \omega_{it} + \xi_{it}$$

and ξ_{it} is distributed according to cdf $F(\cdot)$. Suppose moreover that ω_{it} is commonly known by buyer and sellers at the beginning of time t, whereas ξ_{it} is the buyer's private information.[15]

The case I considered up to now, serially uncorrelated preferences, may be understood as the limit case when $\rho \to 0$. I now focus on the opposite case, namely the case when $\rho \to 1$. I now have to consider a continuum of states, namely the value of ω_t (consumer preference for the incumbent seller). For this reason, I redefine average price as the average across firms and over all possible states in the steady state. Specifically, let $\mu(\omega)$ be the stationary probability density over state ω. Then average price is given by

$$\bar{p} = \int_{-\infty}^{+\infty} \mu(\omega) \sum_{i=0,1} q_i(\omega)\, p_i(\omega)\; d\omega$$

Corollary 1. *There exist ρ' and $\beta'(\rho), \delta'(\rho) \in (0,1)$ such that, if $\rho > \rho'$, $\beta < \beta'(\rho)$ and $\delta < \delta'(d)$, then an increase in s leads to an increase in average price $\bar{p}$.*

Corollary 1 is a corollary of Proposition 5. The latter implies that, if consumers have a strong preference for firm A (over and above the cost of switching) than an increase in s leads to an increase in average price. If $\beta = \delta = 0$ and ρ is close to 1, then, on average, the value of ω in a randomly chosen period is very high. From Proposition 1, this implies that average price is increasing in s.

Doganoglu (2010) and Rhodes (2014) develop models with customer recognition and provide sufficient conditions such that $d\bar{p}/ds < 0$. These results are not inconsistent with Corollary 1 since they refer to a symmetric duopoly. In this sense, they are closer to Proposition 3 than to Corollary 5. I return to this in the next subsection, where I take a more systematic look at the related literature.

An extensive related literature looks at price competition with customer recognition, that is, the situation when consumer preferences are correlated over time and firms can infer such preferences ex-post. However, the central question in this literature is the impact of customer recognition on prices and profits, rather than the impact of switching costs per se. Villas-Boas (1999, 2004), for example, shows that prices and profits are lower than they would be if firms could not recognize consumers.

Bargains and ripoffs: summary and notes on the literature. The central research question in the literature on switching costs is the impact that these costs have on market competition. Most of the literature claims that switching costs are anti-competitive, that is, increase equilibrium prices. However, a number of recent papers have argued the opposite. In this section, I provided a series of results that, together, show that both anti- and pro-competitive effects are possible. Anti-competitive effects of an increase in s emerge when switching costs are large to begin with or when one of the firms is preferred by consumers (independently of switching costs). Pro-competitive effects of an increase in s emerge when switching costs are low and firms are symmetric or when the discount factor is close to 1.

Altogether, these results suggest a general pattern: If the market equilibrium is very competitive to begin with (symmetric oligopoly or high discount factor) then an increase in switching costs makes the market even more competitive. If, by contrast, the market equilibrium is not very competitive to begin with (asymmetric oligopoly as a result of high s or high d) then an increase in s makes the market even less competitive. In other words, switch costs magnify the initial market conditions.

In an infinite game context, Beggs and Klemperer (1992) predict an anti-competitive of switching costs. They assume that each consumer's switching cost is infinite (that is, once they choose a seller they remain locked in for life). While it is not possible to map their model into mine, their result is broadly consistent with my characterization, namely Proposition 5.

[14] Fabra and García (2012) develop a continuous-time dynamic model with switching costs. In their Markov Perfect equilibrium, higher switching costs imply a slower transition to a symmetric market structure and a slower rate of decline for average prices; moreover, if firms are sufficiently asymmetric, an increase in switching costs also leads to higher current prices. However, as market structure becomes more symmetric, price competition turns fiercer and in the long-run, switching costs have a pro-competitive effect.

[15] In the marketing terminology, this case is known as *customer recognition*. Chen and Pearcy (2010) also allow for this possibility. Specifically, they model the joint distribution of consumer preferences over time with a copula to parametrize the degree of temporal dependence in preferences. The focus on their analysis is on the implications of serial correlation for dynamic pricing. Specifically, they find conditions under which firms reward loyal customers. Although their framework allows for the possibility of switching costs, they do not focus on the same questions as the switching costs literature. For additional models with customer recognition, see Chen (1997), Villas-Boas (1999, 2006), Fudenberg and Tirole (2000), Taylor (2003).

Farrell and Shapiro (1988) consider an infinite periods model where, in each period, a new cohort of consumers enters the market. They assume there is no consumer heterogeneity beyond switching costs; and that switching costs are high. As a result, the effect described in Proposition 3 is absent. In fact, they conclude that switching costs increase market price. Similarly, Padilla (1995) and To's (1995) models imply anti-competitive switching costs. Due to the absence of consumer heterogeneity, these models lack the element of competition for new customers that is so central in results such as Proposition 3. (Arie and Grieco, 2014, refer to this as the "compensating" effect, which, as they argue, fosters competitive outcomes.)

By contrast to the older infinite period models, recent papers such as Doganoglu (2010), Rhodes (2014), Fabra and García (2012), show that an increase in switching costs leads to a decrease in prices. Doganoglu's (2010) result is based on the assumption that s increases from zero, so it relates to a particular case of Proposition 3 (particular because it corresponds to the case of small s and because it assumes linear preferences).

Generalizing Doganoglu's analysis, Rhodes (2014) looks at an infinite period model with overlapping generations of two-period lived forward looking consumers. He assumes that firms cannot discriminate between consumers. He finds a "general presumption that in the long-run, switching costs make markets more competitive." However, like Doganoglu (2010), Rhodes (2014) does not consider the possibility of inherent asymmetry regarding preferences for firms, as I do in Proposition 5. Moreover, he assumes linear preferences and caps the value of s. Therefore, while it is difficult to map his model into mine, it seems fair to state that it captures the spirit of the first part of Proposition 3 but not the spirit of the second part of Proposition 3 or of Proposition 5. Rhodes (2014) also makes some important points regarding consumer discounting; I will return to these in Section 5.

Like Rhodes (2014), Fabra and García (2012) focus on the difference between short-run and long-run in a model where firms cannot discriminate between consumers — and where, therefore, market shares matter. Their modeling strategy is different: they consider a continuous-time dynamic model. The conclusion, however, is similar (and consistent with my general characterization): "as market structure becomes more symmetric, price competition turns fiercer and in the long-run, switching costs have a pro-competitive effect."

Pearcy (2011) considers the general n firm case and shows that the effect of switching costs is the more competitive the larger the number of firms. This result is consistent with the overall theme of the present section, namely that switching costs "amplify" market competitiveness, making competitive markets more competitive and non-competitive markets less competitive. (Pearcy, 2011, adds an additional dimension to this result, namely variation in the number of firms.)

In summary, if there is one general statement to make regarding the effect of switching costs on market competitions is that there is no general statement that switching costs are either pro-competitive or anti-competitive. What the theory suggests, however, is that, in markets that are very competitive to begin with, an increase in switching costs increases the level of competition; whereas in markets that are not very competitive to begin with, an increase in switching costs decreases the level of competition.

I conclude with two notes. First, one feature that makes markets competitive is the possibility of discriminating between locked-in and non-locked-in customers. In customer markets, the case I consider in this paper, this is possible by assumption. If sellers cannot discriminate between locked-in and non-locked-in consumers, then one would expect the anti-competitive switching costs with greater likelihood.

Second, I should mention that I restrict to *unilateral effects* of switching costs on market competition. A different question is the impact of switching costs on *coordinated effects*, that is, the likelihood of collusion. Section 6 of McSorley et al. (2003) presents a good discussion of the issue.

4. Additional results

As I mentioned earlier, the core of the paper corresponds to the central question in the switching costs literature, namely the impact that such switching costs have on market competitiveness, namely average price level. The literature includes a number of other issues regarding the effects of switching costs. In this section, I cover three different additional issues: (a) switching costs as an entry barrier; (b) the effect of switching costs on profits and welfare; and (c) endogenously chosen switching costs. Except for the last part in this section, I return to the assumption that ξ is the only source of firm asymmetry (that is, $d = 0$).

Entry barriers. An important policy question (for example, in the telecommunications industry) is whether switching costs make entry more difficult. Intuitively, the idea is that, facing an incumbent with locked-in customers, a potential entrant finds entry prospects less attractive. My model is consistent with this prediction.

Proposition 6. $dv_0/ds < 0$.

In words, the value of a firm that has no customer base is lower the greater the value of s. Suppose, for example, that a potential entrant draws a value of the entry cost ϕ generated by Nature; and that entry takes place if and only if the net profit from entry is positive, that is, $v_0 > \phi$. Proposition 6 then implies that the probability of entry is decreasing in the value of s.

Note that, from firm 0's point of view, the strategic effect of an increase in s may go in the same direction as or in opposite direction to the direct effect. Proposition 6 shows that the total effect is negative. Propositions 2–4 show that, depending on the values of s and δ, an increase in s may lead to an increase or a decrease in the value of p_1. The variation dp_1/ds corresponds to the strategic effect of switching costs (positive or negative strategic effect depending on whether dp_1/ds is positive or negative, respectively). If $dp_1/ds > 0$ (e.g. if δ is sufficiently small), then the direct effect is greater (in absolute value) than the strategic effect: in net terms, firm 0 is worse off as the level of switching costs increases, but it is less worse off than by the direct effect of the switching cost: since an increase in s "softens" the incumbent (harvesting dominates investing), the entrant gets some reprieve from the effect of a higher switching cost.

However, (9) implies that, as $\delta \to 1$ the incumbent's price p_1 is *decreasing* in s. In other words, an increase in s "toughens" the incumbent (investing dominates harvesting), and the entrant's prospects are harmed both by the direct effect and by the strategic effect of an increase in s.

Two additional notes should be made regarding the effect of switching costs on entry. First, while Proposition 6 suggests that a higher s makes entry more difficult, the opposite result is also possible in a model where firms cannot discriminate between locked-in and non-locked-in consumers. When that is the case, the effect of an increase in s may be to soften an incumbent whose "harvesting" incentives lead to higher prices. In terms of the discussion above, there may be cases when the strategic effect of an increase in s is favorable to the entrant and more than outweighs the direct effect (which is unfavorable to the entrant). Farrell and Shapiro (1988) develop a model when this is the case: "in equilibrium the firm with attached customers typically specializes in serving them and concedes new buyers to its rival."[16]

Second, the welfare effects of entry are generally dubious: there may be cases when free entry leads to insufficient entry (namely when entry increases consumer surplus which cannot be captured by the entrant); and there may be cases when free entry leads to excessive entry (namely when the entrant's expected profit form entry corresponds mostly to a transfer from incumbents). For this reason, the welfare effect of an increase in s through changes in entry conditions is in general dubious.[17]

Profits and welfare. So far I have been dealing with the impact of switching costs on average price, one of the central questions in the academic and public policy debate. What can be said about seller profits and consumer welfare? Tentatively, Proposition 3 suggests that under some conditions buyers are better off, and sellers worse off, with switching costs than without; and Propositions 4 and 5 suggest that the opposite may be the case.

However, paying a lower price is only half of the story for a buyer: given private information about preferences, switching actually occurs along the equilibrium path. We must therefore subtract the costs from switching when considering expected buyers surplus. As to the sellers, the fact that average price declines does not imply that sellers are uniformly worse off with switching costs. In fact, as shown earlier, the insider may be able to increase its price as a result of a higher switching cost. In sum, it is not obvious whether switching costs benefit buyers and sellers. My next result provides some answers to these questions.

Proposition 7. *There exists a value $s' > 0$ such that, if $s < s'$, then there exist $\delta'(s)$ and $\delta''(s)$, where $0 < \delta'(s) < \delta''(s) < 1$, such that an increase in switching cost s leads to*

1. *An increase in the insider's value if and only if $\delta < \delta'(s)$*
2. *A decrease in the outsider's value for all δ*
3. *A decrease in industry value (that is, the joint value of insider and outsider) for all δ*
4. *An increase in consumer surplus if and only if $\delta > \delta''(s)$*
5. *A decrease in welfare for all δ*

Notice that part 2 of Proposition 7 repeats Proposition 6 and is included here for completeness. Notice also that, if $\delta'(s) < \delta < \delta''(s)$, then all agents (insider, outsider, buyer) are worse off with switching costs than without.[18]

Earlier, I mentioned that two-period models of switching costs may miss a lot of the important action in infinite-period models; and that the beginning-of-the-world and end-of-the-world effects make it difficult to distinguish the effects of switching costs per se. Regarding welfare analysis, however, the two-period model is basically consistent with Proposition 7. If consumers have unit demand and there is no imperfect information about valuations, then switching costs are welfare neutral: the price increase suffered by consumers in the second period is exactly compensated by a lower price in the first period: bargains and ripoffs are equal in absolute value. However, just about any departure from this extreme case leads to some form of Harberger triangle. For example, if demand is downward sloping then a sequence of bargain then ripoff leads to lower welfare than a sequence of two periods with the same price.

---

[16] See also Wang and Wen (1998).

[17] An additional source of excessive entry is pointed by Farrell and Shapiro (1988): to the extent that sellers cannot discriminate between consumers, an efficient incumbent may let an inefficient entrant into the market.

[18] Biglaiser et al. (2015) also show that an increase in the switching costs of all consumers can lead to a decrease in the profits of the incumbent. However, this takes place in a different context and for different reasons. See also Section 5.

Proposition 7 suggests an interesting question: if switching costs are frequently created by sellers; and if, largely, sellers lose as a result of switching costs; then why do sellers create switching costs? Part of the answer to the question is given by point 1 in Proposition 7: if δ is sufficiently small, then an increase in switching costs increases the value of the incumbent seller. In what follows, I take this issue of step further by considering the possibility that one of the sellers unilaterally increases the cost of switching away from its product.

Endogenous switching cost. I now consider the case when only seller A creates a switching cost. In other words, it costs s_A for a consumer to switch from seller A to seller B, but it costs zero for the consumer to switch from seller B to seller A. Given this asymmetry, I now need to keep track of the seller's identity. Moreover, I need to compute the buyer's value functions explicitly. I continue to use the subscripts 0 and 1 to denote a firm's state (outsider or insider). For notational compactness, I denote by s_i the cost of switching away from firm i, noting that $s_B = 0$.

A buyer who is currently attached to seller i chooses seller i again if and only if

$$\xi_i - p_{1i} + \beta\, u_i \geq -s_i - p_{0j} + \beta\, u_j$$

where u_i is value of being attached to firm i (to be derived below). This implies that the critical values of the buyer's relative preference leading to a switch away from the insider seller are now given by

$$x_i \equiv p_{1i} - p_{0j} - \beta\, u_i + \beta\, u_j - s_i \tag{12}$$

As before, the insider seller's demand is given by $q_{1i} = 1 - F(x_i) = F(-x_i)$, $i = A, B$. As in the treatment of the asymmetric case in the previous section, I need to compute buyer value functions explicitly. Assuming these are measured *before* the buyer learns his valuation parameter ξ, they are recursively given by

$$u_i = \alpha + G(x_i) + G(-x_i) + \big(1 - F(x_i)\big)(-p_{1i} + \delta\, u_i) + F(x_i)(-s_i - p_{0j} + \beta\, u_j)$$

My goal is to evaluate the impact of increasing switching costs (unilaterally, in the present context) on prices and profits.

Proposition 8. *There exists a value s' such that, if $s_A < s'$, then an increase in seller A's switching cost s_A leads to*

1. *A decrease in average price*
2. *An increase in seller A's steady state value*
3. *A decrease in seller B's steady state value*

Notice that point 2 in Proposition 8 is different, and stronger, than point 1 in Proposition 7. The latter refers to an increase in value by the insider seller, whereas the former refers to an increase in value along the steady-state. Consider a seller with an equal number of buyers who are attached to it and buyers who are not attached to it. In this case, Proposition 7 would predict a decline in seller value following a uniform increase in switching costs, whereas Proposition 8 states that, in the steady state, seller value would increase following a unilateral increase in switching cost.

Proposition 8 also suggests that the metagame where sellers choose levels of switching cost has the nature of a prisoner's dilemma. Specifically, consider the following metagame. In a first stage, sellers simultaneously choose $s \in \{s_L, s_H\}$, where $s_H > s_L$ and both s_L and s_H are small. In a second stage, sellers play the infinite period game we have been considering up to now. Then Propositions 3 and 8 imply that the metagame has the structure of a prisoner's dilemma: choosing s_H is a dominant strategy for each seller (by Proposition 8), but the payoff from (s_H, s_H) is lower than the payoff from (s_L, s_L) (by Proposition 3).

This result bears some resemblance to Caminal and Matutes (1990) (see also Banerjee and Summers, 1987), with the important difference that I consider an infinite period price competition model rather than a two-period model.

5. Further extensions

In this section, I discuss the extent to which my results depend on consumer discounting, product differentiation, and price discrimination. I also look at the issue of heterogeneous switching costs as well as the role of market shares.

Rhodes (2014) shows (Lemma 5) that the steady state price strictly decreases in δ and strictly increases in β. This result cannot be exactly mapped into my results: I look at the effect of an increase in β and δ on $d\bar{p}/ds$, not on $\bar{p}$. Having said that, the result regarding δ is broadly consistent with the idea that higher frequency of customer purchases makes the market more competitive (and thus an increase in s makes them even more competitive). Unlike Rhodes (2014), I find that the effect of an increase in β depends on the value of δ. However, my result is derived from the case when firms are asymmetric, whereas Rhodes (2014) considers the symmetric case.

Product differentiation. What is the effect of product differentiation? One natural way of modeling product differentiation is to assume that the distribution of ξ is given by $F(\xi/\sigma)$. In this context, a higher values of σ corresponds to a greater degree of product differentiation. My limit results (small s, large s, large δ, large ρ) do not depend on the shape of $F(x)$ beyond what's assumed in Assumption 1. In this sense, the results do not depend on the degree of product differentiation.

However, I should note that there is an important discontinuity at $\sigma = 0$, that is, there is a important difference between the case when there is a preference shock ξ ("Hotelling" competition) and the case when there isn't one ("Bertrand" competition). If there is no residual product differentiation (the $\sigma = 0$ case), then the insider seller sells with probability 1 regardless of the level of switching cost. By contrast, if the degree of product differentiation is strictly greater than zero, then, as the level of switching costs goes to zero the insider seller sells with probability 50%. This is one of the important differences of my paper (as well as other papers that feature product differentiation) with respect to much of the previous literature: because of residual product differentiation, switching does take place along the equilibrium path.

Uncertain and heterogeneous switching cost. I have assumed throughout that the value of s is known to buyer and sellers. In many real-world situations, the buyer has better information about switching cost than sellers. Suppose that sellers believe that s is distributed according to $G(s)$, whereas the buyer knows the precise value of s. Assume also that s is i.i.d. across periods and independent of buyer valuations ξ_i. Then all of the results in the paper go through if we redefine variables appropriately. First, we let s denote the average of s (according to cdf G). Second, we denote by $F(\xi)$ the convolution of the original $F(\xi)$ and the cdf $G(s)$ shifted by the average of s. In other words, we can include uncertainty about s in the overall uncertainty about valuations ξ_i.[19,20]

A related issue is that of heterogeneity in switching costs, that is, the case when different buyers have different values of the switching cost. To the extent that sellers can discriminate across buyers, the results in the paper still apply. If however discrimination is not possible, then heterogeneity can matter a lot, especially in a dynamic context. This problem lies beyond the scope of the present paper and is discussed in Biglaiser et al. (2015).

Market share and firm value. My assumption of price discrimination implies that I can look at competition for each individual buyer. In aggregate terms, this leads to market share effects similar to those in previous literature. As shown in Proposition 3, a small increase in switching cost implies that the incumbent seller increases its price, whereas the outsider seller decreases its price. Now consider a seller with a market share ν, that is, for a fraction ν this seller is the inside seller. Then we can show that (a) seller average price is linear and increasing in market share; (b) seller value is linear and increasing in market share. Biglaiser et al. (2015) also provide conditions under which firm value is linear and increasing in market share. Finally, under assumptions of symmetry, in the long run market shares tend to 50% (by the law of large numbers), so the larger firm's market share will decrease in expected terms (reversion to the mean). Rhodes (2014) and Fabra and Garcia's (2012) models imply similar dynamics.

6. Conclusion

In a competitive environment, switching costs have two effects. First, switching costs increase the market power of a seller with attached customers. Second, switching costs increase competition for new customers. In this paper, I derived conditions under which switching costs decrease or increase equilibrium prices. Overall, the paper's message is that, if markets are very competitive to begin with, then switching costs make them even more competitive; whereas if markets are not very competitive to begin with, then switching costs make them even less competitive. In the above statements, by "competitive" I mean a market that is close to a symmetric duopoly or one where the sellers' discount factor is very high.

Appendix A

Proof of Lemma 1. First notice that

$$\frac{F(x)^2}{f(x)} = F(x)\,\frac{F(x)}{f(x)}$$

Since $F(x)$ is increasing and $\frac{F(x)}{f(x)}$ is strictly increasing (by Assumption 1), it follows that the product is strictly increasing.

Next notice that, by part (ii) Assumption 1,

$$\frac{F(x)-1}{f(x)} = \frac{-F(-x)}{f(x)} = \frac{-F(-x)}{f(-x)}$$

Since $\frac{F(x)}{f(x)}$ is strictly increasing, $\frac{-F(-x)}{f(-x)}$ is strictly increasing too.

Next notice that

$$\frac{2\,F(x)-1}{f(x)} = \frac{F(x)-1}{f(x)} + \frac{F(x)}{f(x)}$$

[19] This is related to the classical problem of estimating switching costs empirically, namely separating switching costs from consumer heterogeneity.

[20] Francisco Ruiz-Aliseda (private communication) considers the specific case when both the preference shock and the value of switching cost are uniformly distributed. Among other things, he shows that the discontinuity at zero product differentiation, considered in the previous subsection, extends to the case when there is uncertainty about the value of s.

I have just proved that $\frac{F(x)-1}{f(x)}$ is strictly increasing. We thus has the sum of two strictly increasing functions, the result being a strictly increasing function.

Finally, taking the derivative of the fourth expression I get

$$\frac{d}{dz}\left(\frac{\big(1-F(x)\big)^2+\big(F(x)\big)^2}{f(x)}\right)=$$
$$=\frac{\Big(-2\big(1-F(x)\big)f(x)+2\,F(x)\,f(x)\Big)f(x)}{\big(f(x)\big)^2}-\frac{f'(x)\,\Big(\big(1-F(x)\big)^2+\big(F(x)\big)^2\Big)}{\big(f(x)\big)^2}$$
$$=4\left(F(x)-\frac{1}{2}\right)-f'(x)\,\chi$$

where $\chi=\Big(\big(1-F(x)\big)^2+\big(F(x)\big)^2\Big)/\big(f(x)\big)^2$ is positive. The result then follows from Assumption 1. □

Proof of Proposition 1. The seller value functions are given by

$$\begin{aligned} v_1 &= \big(1-F(x)\big)(p_1+\delta\,v_1)+F(x)\,\delta\,v_0 \\ v_0 &= F(x)(p_0+\delta\,v_1)+\big(1-F(x)\big)\,\delta\,v_0 \end{aligned} \tag{13}$$

where δ is the seller's discount factor. The corresponding first-order conditions are

$$\begin{aligned} -f(x)(p_1+\delta\,v_1)+1-F(x)+f(x)\,\delta\,v_0 &= 0 \\ -f(x)(p_0+\delta\,v_1)+F(x)+f(x)\,\delta\,v_0 &= 0 \end{aligned}$$

(Recall that, from (2), $\frac{dx}{dp_1}=1$ and $\frac{dx}{dp_0}=-1$.) Solving for optimal prices, I get

$$\begin{aligned} p_1 &= \frac{1-F(x)}{f(x)}-\delta\,V \\ p_0 &= \frac{F(x)}{f(x)}-\delta\,V \end{aligned} \tag{14}$$

where

$$V\equiv v_1-v_0$$

Substituting (14) for p_1, p_0 in (13) and simplifying, I get

$$v_1=\frac{\big(1-F(x)\big)^2}{f(x)}+\delta\,v_0 \tag{15}$$

$$v_0=\frac{F(x)^2}{f(x)}+\delta\,v_0 \tag{16}$$

It follows that

$$x=\frac{1-F(x)}{f(x)}-\frac{F(x)}{f(x)}-s=\frac{1-2\,F(x)}{f(x)}-s \tag{17}$$

$$V=\frac{\big(1-F(x)\big)^2}{f(x)}-\frac{F(x)^2}{f(x)}=\frac{1-2\,F(x)}{f(x)} \tag{18}$$

Equation (17) may be rewritten as

$$x+\frac{2\,F(x)-1}{f(x)}=-s \tag{19}$$

By Lemma 1, the left-hand side is strictly increasing in x, ranging from $-\infty$ to $+\infty$ as x itself ranges from $-\infty$ to $+\infty$. This implies there exists a unique solution x. From (18), there exists a unique V. Finally, from (14) there exist unique p_0, p_1. □

Proof of Proposition 2. From (19) and Assumption 1, x is strictly decreasing in s. From (14) and $\delta=0$, p_1 is strictly decreasing in x. Since the solution is continuous in δ at $\delta=0$, it follows that an increase in s leads to a decrease in x and thus an increase in p_1. A similar argument holds for p_0. □

Proof of Proposition 3. Lemma 1, (19), and the implicit function theorem imply that x is strictly decreasing in s, ranging from 0 to $-\infty$ as s ranges from 0 to $+\infty$. I will thus consider the derivative of average price with respect to x, from which I then derive the comparative statics with respect to s. Average price is given by

$$\bar{p} \equiv q_1\, p_1 + q_0\, p_0 = \big(1 - F(x)\big)\, p_1 + F(x)\, p_0$$

Substituting (14) for p_1, p_0 and (18) for V, and simplifying, I get

$$\begin{aligned}\bar{p} &= \big(1 - F(x)\big)\left(\frac{1 - F(x)}{f(x)} - \delta\, V\right) + F(x)\left(\frac{F(x)}{f(x)} - \delta\, V\right)\\ &= \frac{\big(1 - F(x)\big)^2}{f(x)} + \frac{F(x)^2}{f(x)} - \delta\, V\\ &= \frac{\big(1 - F(x)\big)^2 + F(x)^2}{f(x)} + \delta\left(\frac{2\,F(x) - 1}{f(x)}\right)\end{aligned} \tag{20}$$

Lemma 1 implies that, at $x = 0$, the first term on the right-hand side of (20) is constant in x; and that the second term on the right-hand side of (20) is increasing in x. It follows that, if x is small, then $\frac{d\bar{p}}{dx} > 0$.

Consider now the case when $x \to -\infty$. Taking the derivative of (20) with respect to x; then taking the limit $x \to -\infty$; and noting that, as $x \to -\infty$, $F(x) \to 0$ and $1 - F(x) \to 1$; we get

$$\lim_{x\to-\infty} \frac{\partial}{\partial x}\left(\frac{\big(1 - F(x)\big)^2 + F(x)^2}{f(x)} + \delta\left(\frac{2\,F(x) - 1}{f(x)}\right)\right) = -\left(2 + \lim_{x\to-\infty} \frac{f'(x)}{f(x)^2}\right)(1 - \delta) < 0$$

where I use the fact that, as $x \to -\infty$, $f'(x) > 0$. It follows that, if x is sufficiently close to $-\infty$, then $\frac{d\bar{p}}{dx} < 0$. □

Proof of Proposition 4. From (20),

$$\lim_{\delta\to1} \bar{p} = 2\,\frac{F(x)^2}{f(x)}$$

Lemma 1 then implies that, if δ is sufficiently close to 1, then $\bar{p}$ is increasing in x. From the proof of Proposition 3, x is decreasing in s. (Notice that that statement does not depend on s being small.) The result then follows by the chain rule of differentiation. □

Proof of Proposition 5. Value functions and first-order conditions parallel the derivation in the proof of Proposition 3 (starting with (13)), only that now value functions and prices are indexed by seller identity. Notice that, as $d \to \infty$, $x_A \to -\infty$ and $x_B \to +\infty$. To see why, suppose that x_A and x_B remain bounded while $d \to \infty$. Then from (14) all prices are bounded; whereas from (11) all firm value functions are bounded. But then (10) implies that the values of x_i are unbounded, which contradicts the hypothesis that they are bounded. This implies that, in the limit as $d \to \infty$, $F(x_A) \to 0$ and $F(x_B) \to 1$.

In what follows, I use the notation, for a generic variable x,

$$\dot{x} \equiv \lim_{d\to\infty} \frac{dx}{ds}$$

Taking derivatives of (14) (with prices indexed by firm identity) with respect to s and then limits as $d \to \infty$ I get

$$\begin{aligned}\dot{p}_{1A} &= -\dot{x}_A - 2\,\delta\,\dot{x}_A + 2\,\delta\,\dot{x}_B\\ \dot{p}_{0A} &= +\dot{x}_B - 2\,\delta\,\dot{x}_A + 2\,\delta\,\dot{x}_B\\ \dot{p}_{1B} &= -\dot{x}_B\\ \dot{p}_{0B} &= +\dot{x}_A\end{aligned} \tag{21}$$

Taking derivatives of (11) with respect to s and then limits as $d \to \infty$ I get

$$\begin{aligned}\dot{u}_A &= -\dot{p}_{1A} + \beta\,\dot{u}_A\\ \dot{u}_B &= -1 - \dot{p}_{0A} + \beta\,\dot{u}_A\end{aligned} \tag{22}$$

where I define $g(x) \equiv dG(x)/dx$ and note that $\lim_{x_i\to\infty} g(x_i) = \lim_{x_i\to\infty} g(-x_i) = 0$. Taking derivatives of (10) with respect to s and then limits as $d \to \infty$ I get

$$\begin{aligned}\dot{x}_A &\equiv \dot{p}_{1A} - \dot{p}_{0B} - \beta\,(\dot{u}_A - \dot{u}_B) - 1\\ \dot{x}_B &\equiv \dot{p}_{1B} - \dot{p}_{0A} + \beta\,(\dot{u}_A - \dot{u}_B) - 1\end{aligned} \tag{23}$$

The system formed by (21), (22) and (23) includes 8 equations and 8 unknowns. Solving form $\dot{p}_{1A}$, I get

$$\dot{p}_{1A} = \frac{1}{3}\,(1 - 2\,\delta)\,(1 + \beta)$$

Since $q_{iA} \to 1$ as $d \to \infty$, average price is determined by p_{1A}, whereas changes in average price are determined by $\dot{p}_{1A}$. The result follows. □

Proof of Proposition 5. Value functions and first-order conditions parallel the derivation in the proof of Proposition 3, only that now value functions and prices are indexed by seller identity. Specifically, the seller value functions are given by

$$\begin{aligned} v_{1i} &= \big(1 - F(x_i)\big)(p_{1i} + \delta\, v_{1i}) + F(x_i)\,\delta\, v_{0i} \\ v_{0i} &= F(x_i)(p_{0i} + \delta\, v_{1i}) + \big(1 - F(x_i)\big)\,\delta\, v_{0i} \end{aligned} \tag{24}$$

$i = A, B$. The corresponding first-order conditions are

$$-f(x_i)(p_{1i} + \delta\, v_{1i}) + 1 - F(x_i) + f(x_i)\,\delta\, v_{0i} = 0$$
$$-f(x_i)(p_{0i} + \delta\, v_{1i}) + F(x_i) + f(x_i)\,\delta\, v_{0i} = 0$$

Solving for optimal prices, I get

$$\begin{aligned} p_{1i} &= \frac{1 - F(x_i)}{f(x_i)} - \delta\,(v_{1i} - v_{0i}) \\ p_{0i} &= \frac{F(x_i)}{f(x_i)} - \delta\,(v_{1i} - v_{0i}) \end{aligned} \tag{25}$$

It follows from (10) that

$$x_i = \frac{1 - 2\,F(x_i)}{f(x_i)} - \beta\,(u_i - u_j) - s - d_i \tag{26}$$

where u_i is given by (11). Substituting (25) for p_{1i}, p_{0i} in (24) and simplifying, I get

$$\begin{aligned} v_{1i} &= \frac{\big(1 - F(x_i)\big)^2}{f(x_i)} + \delta\, v_{0i} \\ v_{0i} &= \frac{F(x_i)^2}{f(x_i)} + \delta\, v_{0i} \end{aligned} \tag{27}$$

At $\beta = \delta = 0$, (26) turns into

$$x_i + \frac{2\,F(x_i) - 1}{f(x_i)} = -s - d_i \tag{28}$$

Assumption 1 implies that the left-hand side is strictly increasing in x_i, ranging from $-\infty$ to $+\infty$ as x_i itself ranges from $-\infty$ to $+\infty$. It follows that there exists a unique value of x_i solving the equation. Moreover, it is strictly decreasing in s and in d_i. This implies that, as $d \to \infty$, $x_A \to -\infty$ and $x_B \to +\infty$. Next, $x_A \to -\infty$ as $d \to \infty$ implies $q_{iA} \to 1$ as $d \to \infty$. This in turn implies that the steady-state average price is determined by p_{1A}.

From (25), we get (at $\delta = 0$)

$$p_{1A} = \frac{1 - F(x_A)}{f(x_A)}$$

The right-hand side is strictly decreasing in x_A. From (28), x_A is strictly decreasing in s. It follows that p_{1A} is strictly increasing in s.

The second part of the proof examines the equilibrium in the neighborhood of $\beta = \delta = 0$. Consider the system of equations producing the equilibrium. This is given by the price equations (25), the demand threshold equations (26), the firm value functions (27), and the consumer value functions (11). Let

$$\mathbf{z} \equiv (p_{1A}, p_{0A}, p_{1B}, p_{0B}, x_A, x_B, v_{1A}, v_{0A}, v_{1B}, v_{0B}, u_A, u_B)$$

be the corresponding vector of equilibrium variables. We thus have a system of 12 equations and 12 unknowns. Represent this system as $f_i(\mathbf{z}; \beta, \delta) = 0$. The matrix of partial derivatives $\partial z_i / \partial z_j$ at $\beta = \delta = 0$, which I denote $\triangledown f$, is a block matrix given by:

$$\triangledown f = \begin{bmatrix} I & A & 0 & 0 \\ 0 & B & 0 & 0 \\ 0 & C & I & 0 \\ 0 & D & 0 & I \end{bmatrix} \tag{29}$$

where A, B, C, D include finite values. The Laplace Expansion Theorem implies that

$$\det \begin{pmatrix} M_1 & 0 \\ M_2 & M_3 \end{pmatrix} = \det(M_1)\,\det(M_3) \tag{30}$$

Applying (30) repeatedly to (29), we conclude that $\det(\triangledown f) = \det(B)$. Matrix B includes finite, non-zero elements in the main diagonal, zeros otherwise. We conclude that $\det(B) \neq 0$, and thus that $\triangledown f$ has full rank. Moreover, all f_i are continuously differentiable with respect to all variables z_i as well as β and δ. Therefore, the Implicit Function Theorem applies, which implies that there exists a unique equilibrium in the neighborhood of $\mathbf{z}^*$ and $\beta = \delta = 0$, where $\mathbf{z}^*$ is the equilibrium at $\beta = \delta = 0$; and that this equilibrium is continuous in β, δ.

Finally, since average price is strictly increasing in s, the result follows by continuity around $\beta = \delta = 0$. □

Proof of Corollary 1. Suppose that $\beta = \delta = 0$. Then in each period the equilibrium corresponds to Proposition 5, where $d = \omega$ if $\omega > 0$ and $d = -\omega$ if $\omega < 0$. It follows that there exists a ω' such that average price is strictly increasing in s when $\omega > \omega'$. The variance of ω tends to ∞ as $\rho \to 1$. Therefore, the probability that $|\omega| > \omega'$ tends 1 as $\rho \to 1$. Since the derivative $d\bar{p}/ds$ is strictly positive for $\omega > \omega'$, there exists a ρ' such that, averaging over all ω, $d\bar{p}/ds$. Finally, the result follows by the same continuity argument as in Proposition 5. □

Proof of Proposition 6. From (19), x is decreasing in s. From (16), v_0 is increasing in x. □

Proof of Proposition 7. From the proof of Proposition 1, we know that

$$\begin{aligned} v_1 &= \frac{\big(1 - F(x)\big)^2}{f(x)} + \delta\, v_0 \\ v_0 &= \frac{F(x)^2}{f(x)} + \delta\, v_0 \end{aligned} \tag{31}$$

where the value of x is given by

$$x + \frac{2\,F(x) - 1}{f(x)} = -s \tag{32}$$

By Assumption 1 and Lemma 1, the left-hand side of (32) is increasing in x. It follows that, by the implicit function theorem, x is decreasing in s. In particular $\partial x / \partial s|_{s=0} = -\frac{1}{3}$.

Lemma 1 also implies that $F(x)^2/f(x)$ is increasing in x and $(1 - F(x))^2/f(x)$ decreasing in x. From (31), this implies that v_0 is decreasing in x, whereas v_1 is increasing in x if and only if δ is sufficiently low. Specifically, at $s = 0$ we have

$$\begin{aligned} \frac{\partial v_0}{\partial s} &= \frac{1}{1-\delta}\,\frac{\partial x}{\partial s} \\ \frac{\partial v_1}{\partial s} &= \left(1 + \delta\,\frac{-1}{1-\delta}\right)\frac{\partial x}{\partial s} = -\frac{1 - 2\,\delta}{1-\delta}\,\frac{\partial x}{\partial s} \\ \frac{\partial (v_0 + v_1)}{\partial s} &= \frac{2\,\delta}{1-\delta}\,\frac{\partial x}{\partial s} \end{aligned}$$

Since $\partial x / \partial s < 0$, we conclude that, at $s = 0$, both v_0 and $v_0 + v_1$ are decreasing in s, whereas v_1 is increasing in s if and only if $\delta < \frac{1}{2}$.

Next consider consumer welfare. Per period expected consumer surplus is given by

$$u = \alpha + G(x) + G(-x) - \big(1 - F(x)\big)\,p_1 - F(x)\,(p_0 + s) \tag{33}$$

Differentiating (33) with respect to s, we get

$$\frac{\partial u}{\partial s} = \Big(g(x) - g(-x) + f(x)\,(p_1 - p_0 - s)\Big)\,\frac{\partial x}{\partial s} - \big(1 - F(x)\big)\,\frac{\partial p_1}{\partial s} - F(x)\,\frac{\partial p_0}{\partial s} - F(x)$$

Evaluating at $s = 0$, we get

$$\left.\frac{\partial u}{\partial s}\right|_{s=0} = -\frac{\partial \bar{p}}{\partial s} - \frac{1}{2}$$

Differentiating (19) we get $\partial x/\partial s = -\frac{1}{3}$. Differentiating (20), we get $\frac{\partial \bar{p}}{\partial x} = 2\,\delta$. It follows that

$$\left.\frac{\partial u}{\partial s}\right|_{s=0} = \frac{2}{3}\,\delta - \frac{1}{2}$$

It follows that consumer surplus increases if and only if $\delta > \frac{3}{4}$. Finally, the result regarding total welfare is trivial: since the market is covered, all price effects are simply a transfer between buyers and sellers. The net effects on consumer welfare come from "transportation cost" (an effect of second order at $s = 0$) and switching costs (a first-order effect).[21] This implies that an increase in s has a first-order negative effect on total welfare. □

Proof of Proposition 8. In what follows, I use the notation, for a generic variable x,

$$\hat{x} \equiv \left.\frac{dx}{ds_A}\right|_{s_A=0}$$

Note that, at $s_A = 0$, we have a symmetric outcome where $x_A = x_B = 0$, $u_A = u_B = u$, and $p_{1A} = p_{0B} = p_{0A} = p_{1B} = p$. Differentiating the buyer value functions with respect to s_A at $s_A = 0$ and defining $g(x) \equiv \frac{dG(x)}{dx}$, I then get

$$\begin{aligned}\hat{u}_A &= g(0)\,\hat{x}_A - g(0)\,\hat{x}_A + \frac{1}{2}\,(-\hat{p}_{1A} + \beta\,\hat{u}_A) - f(0)\,\hat{x}_A(-p + \beta\,u) + \\ &\quad + \frac{1}{2}\,(-1 - \hat{p}_{0B} + \beta\,\hat{u}_B) + f(0)\,\hat{x}_A(-p + \beta\,u) \\ &= \frac{1}{2}(\beta\,\hat{u}_A + \beta\,\hat{u}_B - \hat{p}_{1A} - \hat{p}_{0B} - 1) \\ \hat{u}_B &= \frac{1}{2}(\beta\,\hat{u}_A + \beta\,\hat{u}_B - \hat{p}_{1B} - \hat{p}_{0A})\end{aligned} \tag{34}$$

This is intuitive: a buyer's expected valuation increases by the increase in future expected valuation, $\delta\,\frac{1}{2}\,(\hat{u}_A + \hat{u}_B)$, minus the increase in expected price paid this period, which is given by $\frac{1}{2}\,(\hat{p}_{1A} + \hat{p}_{0B})$ if the buyer is attached to seller A and $\frac{1}{2}(\hat{p}_{1B} + \hat{p}_{0A})$ if the buyer is attached to seller B. Moreover, if the buyer is attached to seller A, buyer welfare further decreases by an additional $\frac{1}{2}\,s$, the probability that an immediate switch to seller B will take place.

Differentiating (12) with respect to s_A at $s_A = 0$, I get

$$\begin{aligned}\hat{x}_A &= \hat{p}_{1A} - \hat{p}_{0B} - \beta\,(\hat{u}_A - \hat{u}_B) - 1 \\ \hat{x}_B &= \hat{p}_{1B} - \hat{p}_{0A} - \beta\,(\hat{u}_B - \hat{u}_A)\end{aligned} \tag{35}$$

The derivation of value functions and first-order conditions is identical to those in the proof of Proposition 1, leading to (14), with the difference that the values of x_i are now different. Differentiating with respect to s_A at $s_A = 0$, and noting that $f'(0) = 0$, I get

$$\begin{aligned}\hat{p}_{1i} &= -(1-\delta)\,\hat{x}_i + \delta\,\hat{x}_j \\ \hat{p}_{0i} &= \delta\,\hat{x}_i + (1+\delta)\,\hat{x}_j\end{aligned} \tag{36}$$

The system formed by (34), (35) and (36) includes 8 equations and 8 unknowns. Its solution is given by

$$\begin{aligned}\hat{p}_{1A} &= \frac{1}{3}\Big(1 - \delta\,(1-\beta) - \beta/2\Big) \\ \hat{p}_{0A} &= -\frac{1}{3}\Big(\delta\,(1-\beta) - \beta/2\Big) \\ \hat{p}_{1B} &= -\frac{1}{3}\Big(\delta\,(1-\beta) + \beta/2\Big) \\ \hat{p}_{0B} &= -\frac{1}{3}\Big(1 + \delta\,(1-\beta) - \beta/2\Big)\end{aligned} \tag{37}$$

Recall that these are variations with respect to the equilibrium values at $s_A = 0$. The above values indicate that seller A, by creating a switching cost s_A, is able to increase its price when the buyer is locked-in, specifically by $\frac{1}{2}\,(1-\delta)ds_A$. If the buyer is locked-in to seller B, however, then seller A must decrease its price by $\frac{\delta}{2}\,ds_A$.

[21] By analogy with the Hotelling model, I refer to "transportation cost" as the utility lost from purchasing a good that is more "distant" from a consumer's "address" than the alternative, that is, the utility lost from purchasing a good with $z_i < 0$.

In a steady state and with $s_A = 0$, all prices are paid by the consumer with equal probability. It follows that

$$\frac{\partial \bar{p}}{\partial s} = -\frac{1}{3}\,\delta\,(1-\beta) \tag{38}$$

Differentiating (15)–(16) (with vs indexed by firm identity), we get

$$\hat{v}_{1i} = -\hat{x}_i + \delta\,\hat{v}_{0i}$$
$$\hat{v}_{0i} = \hat{x}_j + \delta\,\hat{v}_{0i}$$

Substituting (37) for $\hat{x}_i, \hat{x}_j$ and solving, we get

$$\hat{v}_{1A} = \gamma\left(1 + (1-\beta)\,(1-2\,\delta)\right)$$
$$\hat{v}_{0A} = \gamma\,\beta$$
$$\hat{v}_{1B} = -\gamma\left(1 - (1-\beta)\,(1-2\,\delta)\right)$$
$$\hat{v}_{0B} = -\gamma\,(2-\beta)$$

where $\gamma \equiv \frac{1}{6\,(1-\delta)}$.

At $s_A = 0$, both states are visited with equal probability. It follows that steady state values are given by

$$\hat{\hat{v}}_A = \frac{1}{2}(\hat{v}_{0A} + \hat{v}_{1A}) = \frac{1}{6}\left(1 + \beta\,\frac{\delta}{1-\delta}\right)$$
$$\hat{\hat{v}}_B = \frac{1}{2}(\hat{v}_{0B} + \hat{v}_{1B}) = -\frac{1+\delta\,(1-\beta)}{6\,(1-\delta)}$$
$$\hat{\hat{v}} = \frac{1}{4}(\hat{v}_{0A} + \hat{v}_{1A} + \hat{v}_{0B} + \hat{v}_{1B}) = -\frac{\delta\,(1-\beta)}{6\,(1-\delta)} \tag{39}$$

Equations (38) and (39) imply the result. □

References

Arie, Guy, Grieco, Paul L.E., 2012. Do Firms Compensate Switching Consumers? Kellogg and Penn State.

Arie, Guy, Grieco, Paul, 2014. Who pays for switching costs? Quantitative Marketing and Economics 12, 379–419.

Banerjee, A., Summers, L., 1987. On frequent flyer programs and other loyalty-inducing arrangements. HIER DP # 1337.

Beggs, Alan W., Klemperer, Paul, 1992. Multi-period competition with switching costs. Econometrica 60, 651–666.

Biglaiser, G., Crémer, J., Dobos, G., 2015. The value of switching costs. Journal of Economic Theory. forthcoming.

Cabral, Luís, 2009. Small switching costs lead to lower prices. Journal of Marketing Research 46 (4), 449–451.

Cabral, Luís, Villas-Boas, J. Miguel, 2005. Bertrand supertraps. Management Science 51, 599–613.

Caminal, Ramon, Matutes, Carmen, 1990. Endogenous switching costs in a duopoly model. International Journal of Industrial Organization 8, 353–373.

Chen, Yongmin, 1997. Paying customers to switch. Journal of Economics & Management Strategy 6 (4), 877–897.

Chen, Yongmin, Pearcy, Jason, 2010. Dynamic pricing: when to entice brand switching and when to reward consumer loyalty. The RAND Journal of Economics 41 (4), 674–685.

Doganoglu, Toker, 2010. Switching costs, experience goods and dynamic price competition. Quantitative Marketing and Economics 8 (2), 167–205.

Dubé, Jean-Piere, Hitsch, Günter J., Rossi, Peter E., 2009. Do switching costs make markets less competitive? Journal of Marketing Research 46, 435–445.

Fabra, Natalia, García, Alfredo, 2012. Dynamic price competition with switching costs. CEPR Discussion paper 8849.

Farrell, Joseph, Klemperer, Paul, 2007. Coordination and lock-in: competition with switching costs and network effects. In: Armstrong, M., Porter, R. (Eds.), Handbook of Industrial Organization, vol. 3. North Holland Publishing, Amsterdam.

Farrell, Joseph, Shapiro, Carl, 1988. Dynamic competition with switching costs. The RAND Journal of Economics 19, 123–137.

Farrell, Joseph, Shapiro, Carl, 1989. Optimal contracts with lock-in. The American Economic Review 79 (1), 51–68.

Fudenberg, Drew, Tirole, Jean, 2000. Customer poaching and brand switching. The RAND Journal of Economics 31, 634–657.

Klemperer, Paul, 1987a. Markets with consumer switching costs. The Quarterly Journal of Economics 102 (2), 375–394.

Klemperer, Paul, 1987b. The competitiveness of markets with switching costs. The RAND Journal of Economics 18 (1), 138–150.

Klemperer, Paul, 1995. Competition when consumers have switching costs: an overview with applications to industrial organization, macroeconomics, and international trade. Review of Economic Studies 62, 515–539.

McSorley, Ciara, Padilla, Atilano Jorge, Williams, Mark, Fernandez, Daniel, Reyes, Tatiana, 2003. Switching costs. A report prepared for the Office of Fair Trading and the Department of Trade and Industry by National Economic Research Associates.

Padilla, A. Jorge, 1992. Mixed pricing in oligopoly with consumer switching costs. International Journal of Industrial Organization 10 (3), 393–411.

Padilla, A. Jorge, 1995. Revisiting dynamic duopoly with consumer switching costs. Journal of Economic Theory 67, 520–530.

Pearcy, Jason, 2011. Bargains Followed by Bargains: When Switching Costs Make Markets More Competitive. Montana State University.

Rhodes, Andrew, 2014. Re-examining the effects of switching costs. Economic Theory 57, 161–194.

Shaffer, G., Zhang, Z., 2000. Pay to switch or pay to stay: preference-based price discrimination in markets with switching costs. Journal of Economics & Management Strategy 9 (3), 397–424.

Shin, J., Sudhir, K., 2009. Switching costs and market competitiveness: de-constructing the relationship. Journal of Marketing Research 46 (4), 446–449.

Taylor, Curtis, 2003. Supplier surfing: competition and consumer behavior in subscription markets. The RAND Journal of Economics 34, 223–246.

To, Theodore, 1995. Multiperiod competition with switching costs: an overlapping generations formulation. Journal of Industrial Economics 44, 81–87.

Villas-Boas, J. Miguel, 1999. Price competition with customer recognition. The RAND Journal of Economics 30, 604–631.

Villas-Boas, J. Miguel, 2004. Price cycles in markets with customer recognition. The RAND Journal of Economics 35 (3), 486–501.
Villas-Boas, J. Miguel, 2006. Dynamic competition with experience goods. Journal of Economics & Management Strategy 15, 37–66.
von Weizsacker, C.C., 1984. The costs of substitution. Econometrica 52, 1085–1116.
Wang, R., Wen, Q., 1998. Strategic invasion in markets with switching costs. Journal of Economics & Management Strategy 7 (4), 521–549.