Dynamic Price Competition with Network Effects Appendix: Numerical Results

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The model presented in the main text does not admit a general analytical solution. In this appendix I proceed to derive the equilibrium numerically and compute it for a variety of values in the parameter space.[1]

In what follows, I assume that $\Phi(x)$ is a standardized Normal. Moreover, I assume that aftermarket payoffs are given by

$$\lambda(i) = (1-\alpha)\,\psi\, i$$
$$\theta(i) = \alpha\,\psi\, i^2$$

One advantage of this parameterization is that, by changing the value of α from 0 to 1, I can consider the extreme cases when all of the aftermarket value is captured by consumers ($\alpha = 0$) or by firms ($\alpha = 1$), while keeping total aftermarket value constant (that is, $i\,\lambda(i) + j\,\lambda(j) + \theta(i) + \theta(j)$ is held constant for each i). Finally, for simplicity I consider $\eta = 101$. Recall that the state space is given by $\{0, \dots, \eta - 1\}$, in this case $\{0, \dots, 100\}$.

The main parameters of interest are therefore: δ, the discount factor; ψ, the degree of network effects; and α, the degree to which network effects are

1. The Gaussian method I use is fairly standard and will be furnished upon request. Important references on such numerical methods include Doraszelski and Pakes (2007) and Doraszelski and Satterthwaite (2009).

captured by firms.[2]

■ **Discount factor (δ).** Let us first consider the effect of changing the value of δ. Proposition 3 pertains to the case when the value of δ is small. It states that, if network benefits are solely received by consumers, then $p(i)$ is increasing, whereas if network benefits are solely received by firms then the opposite is true. Does this characterization extend to higher values of δ? Figure 1 shows equilibrium prices for various values of δ. The left-hand panels (case A) correspond to the case when $\theta(i) = 0$ and $\lambda(i) = \psi\ \frac{i}{\eta}$, that is, all network benefits are captured by consumers ($\alpha = 0$). The right-hand panels (case B) correspond to the case when $\theta(i) = \psi\ \frac{i^2}{\eta}$ and $\lambda(i) = 0$, that is, all network benefits are captured by firms ($\alpha = 1$).

Solid lines correspond to $\delta = 0$. As can be seen, $p(i)$ is increasing in case A and decreasing in case B, as predicted by Proposition 3. In fact, this pattern holds true for higher values of δ, e.g., $\delta = .6$. However, for very high values of δ we see that, in case B, $p(i)$ becomes increasing for high values of i. To understand the factors underlying these patters, it is useful to recall that $p(i) = h(i) - w(i)$, where $h(i)$ is associated to short-run market power and $w(i)$ denotes the value, in terms of future payoffs, of winning the current sale. When $\theta(i) = 0$ (left panels), the value of winning a sale (attracting a new consumer), in terms of future payoff, is relatively low compared to the revenue obtained from that new consumer. In other words, the value of $h(i)$ dominates the value of $w(i)$. Since $h(i)$ is increasing in i, we obtain $p(i)$ also increasing in i.

When $\theta(i)$ is "convex," however (right panels), then the value of winning a new consumer is increasing in i and becomes significantly larger as δ increases; but so does the value of $h(i)$. We thus have a "race" between "harvesting," the incentive to exploit market power and price higher, and "investing," the incentive to price low so as to maintain a high market share. The result of this "race" is that, for high values of δ, $p(i)$ becomes U shaped. As can be seen from the right-hand panels in Figure 1, the reason for $p(i)$ to become U shaped is that $h(x)$ is convex.

Convexity of $h(x)$ depends importantly on the primitive functional forms

2. I could also change the degree of product differentiation by assuming that the variance of x is given by σ and considering values of σ different from 1. However, by an appropriate change of units, I can normalize $\sigma = 1$. In other words, the value of ψ should be interpreted as the degree of network effects relative to degree of product differentiation.

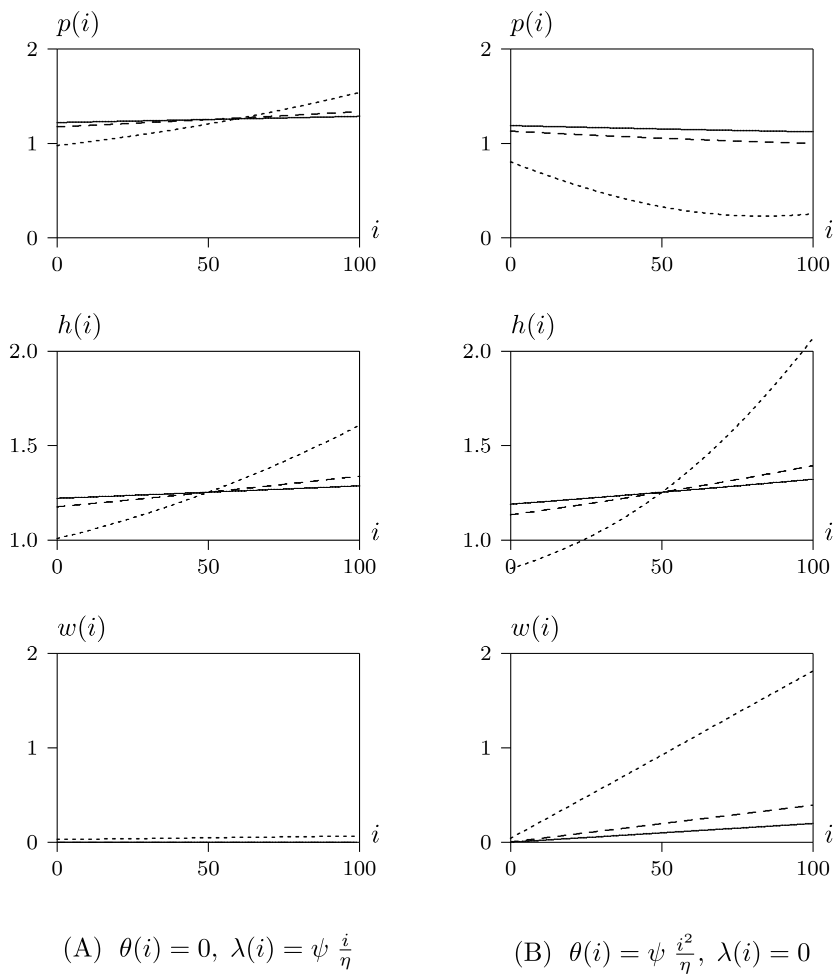


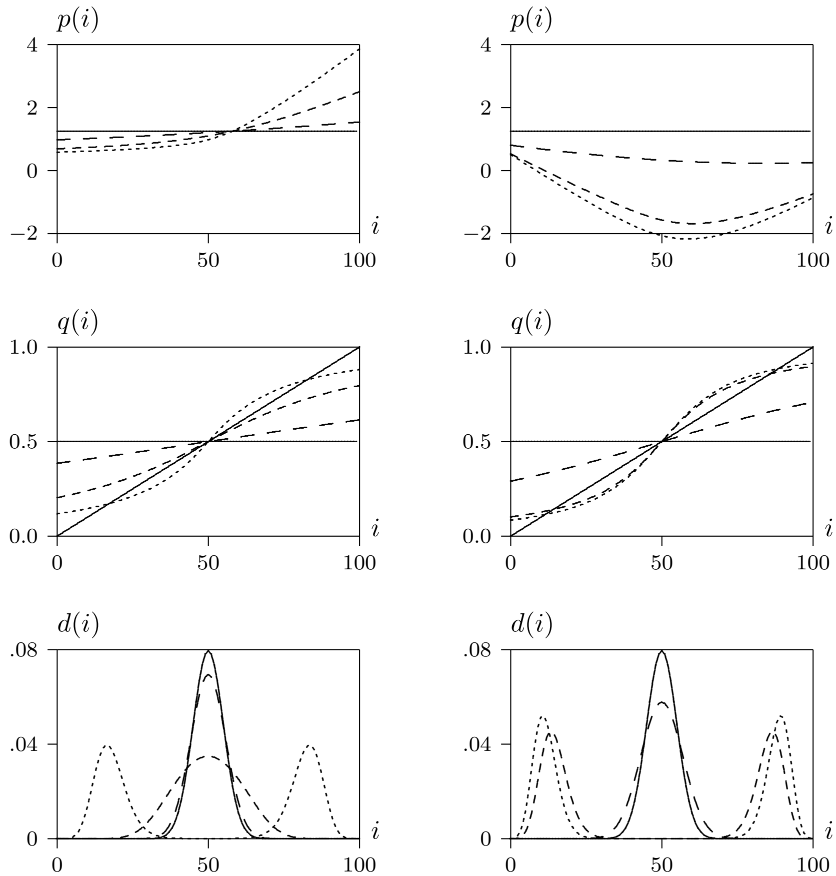
Figure 1: Effects of changing δ. Values of δ are 0, 0.6 and 0.9 (solid, dashed and dotted lines, respectively). In all cases, $\psi = .1$

of the model. In fact, $h(i) \equiv q(i)/q'(i)$, where $q(i) = 1 - \Phi\left(x(i)\right)$. If $\Phi(\cdot)$ is a Normal distribution, as I am assuming, then $\left(1 - \Phi(x)\right)/\phi(x)$ is not only increasing (as implied by Assumption 1) but also convex. Convexity also holds for other distributions, such as the lognormal or the student or the Cauchy, but not all: for example, it does not hold for the uniform. It thus appears that, beyond the characterization provided by Proposition 3, there isn't much that can be said about the shape of $p(i)$.

■ **Strength of network effects (ψ).** Figure 2 describes the effect of changing the value of ψ, the parameter measuring the intensity of network effects. As before, I distinguish between the extremes when all aftermarket network benefits accrue to consumers ($\alpha = 0$, left panels) and when all aftermarket network benefits accrue to firms ($\alpha = 1$, right panels). The patterns of pricing (top panels) are consistent with those obtained in Figure 1, where we consider different values of the discount factor. In particular, $p(i)$ is increasing when $\alpha = 0$ and decreasing or U shaped when $\alpha = 1$. Consistently with Proposition 5, we observe that the pricing function is approximately linear when ψ is small. However, higher values of ψ lead to a convex pricing function. Moreover, when $\alpha = 0$, as the degree of network effects increases, small networks decrease their price and large networks increase their price. By contrast, when $\alpha = 1$, an increase in ψ uniformly leads to a decrease in prices.

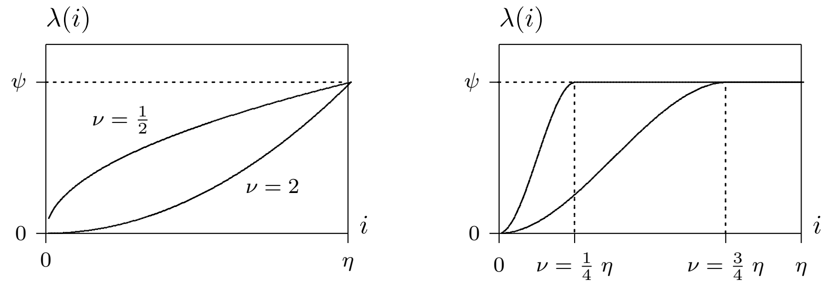
The middle set of panels in Figure 2 show the effect of increasing ψ on $q(i)$, the probability that a newborn consumer joins network i. For small values of ψ (e.g., $\psi = .1$), $q(i)$ is approximately linear, as predicted by Proposition 5. Higher values of ψ, however, are associated with an S shaped $q(i)$ function; and the greater ψ is, the steeper the middle section of the S is. This is consistent with Proposition 4, which states that, if $\lambda(i)$ is sufficiently steep or $\theta(i)$ sufficiently convex, then, around i^* (in the present case, $i^* = 50$), market share dynamics are characterized by strong market dominance, that is, the large network's birth rate is greater than its death rate, and so market shares move away from symmetry (in expected value).

Finally, the bottom panels in Figure 2 depict the stationary distribution of market shares as a function of ψ. For small values of ψ, the distribution of market shares in unimodal and centered around $i^* = 50$. For higher values, however, it becomes bimodal. Specifically, if $q(i)$ (birth rate) crosses the diagonal (death rate) from below at i^*, then we obtain a bimodal stationary



(A) $\theta(i) = 0,\ \lambda(i) = \psi\ \frac{i}{\eta}$ (B) $\theta(i) = \psi\ \frac{i^2}{\eta},\ \lambda(i) = 0$

Figure 2: Effects of changing ψ. Values of ψ are 0, 0.1, 0.3 and 0.5 (solid, long-dashed, short-dashed and dotted lines, respectively). In all cases, $\delta = .9$



(A) $\lambda(i) = \psi \left(\frac{i}{\eta}\right)^{\nu}$ (B) $\lambda(i) = \psi \left(\frac{i}{\nu}\right)^2 \left(3 - 2\,\frac{i}{\nu}\right) \quad (i \le \nu)$

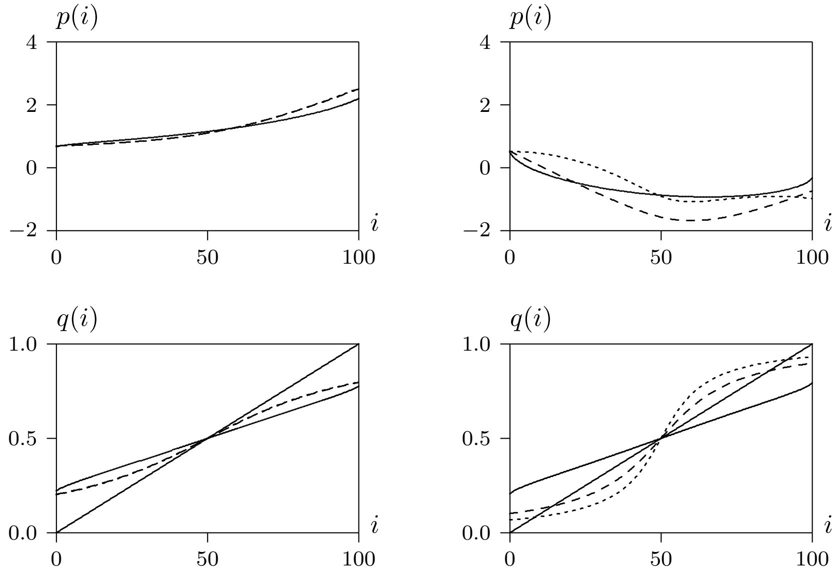
Figure 3: Alternative functional forms of benefit function $\lambda(i)$. In case (A), $\lambda(i)$ is concave if and only if $\nu \le 1$. In case (B), $\lambda(i)$ plateaus at ν.

distribution. Moreover, the modes of such bimodal distribution are given by the points where $q(i)$ crosses the diagonal from above.

■ **Aftermarket payoff functional forms (λ, θ).** All of the above numerical simulations were performed assuming a linear consumer benefit function $\lambda(i)$. This was done for convenience and because a linear function is a natural starting point. However, there is no theoretical reason for such functional form. There are at least two ways in which we can extend the analysis. One is to consider a fixed exponent benefit function (with the linear function being a particular case). A second one is to consider a benefit function that plateaus at some market share level. Figure 3 depicts the benefit function $\lambda(i)$ in these two cases. The left panel depicts the case when $\lambda(i) = \psi \left(\frac{i}{\eta}\right)^{\nu}$ and ν takes on the values $\frac{1}{2}$ (concave benefit function) and 2 (convex benefit function). The right panel depicts the case when $\lambda(i) = \psi \left(\frac{i}{\nu}\right)^2 \left(3 - 2\,\frac{i}{\nu}\right)$ and ν takes on the values $\frac{1}{4}\,\eta$ (low critical mass) and $\frac{3}{4}\,\eta$ (high critical mass).[3] As in my previous simulations, I consider separately the cases when $\alpha = 0$ and $\alpha = 1$. Moreover, in the latter case, I choose $\theta(i)$ so as to maintain total aftermarket value constant with respect to the $\alpha = 0$ case.

Figure 4 presents results for the case when $\lambda(i) = \psi \left(\frac{i}{\eta}\right)^{\nu}$. The top pan-

3. The second functional form is the lowest polynomial $\lambda(i)$ with the properties that the derivative of benefit with respect to network size is zero at $i = 0$ and $i = \nu$.

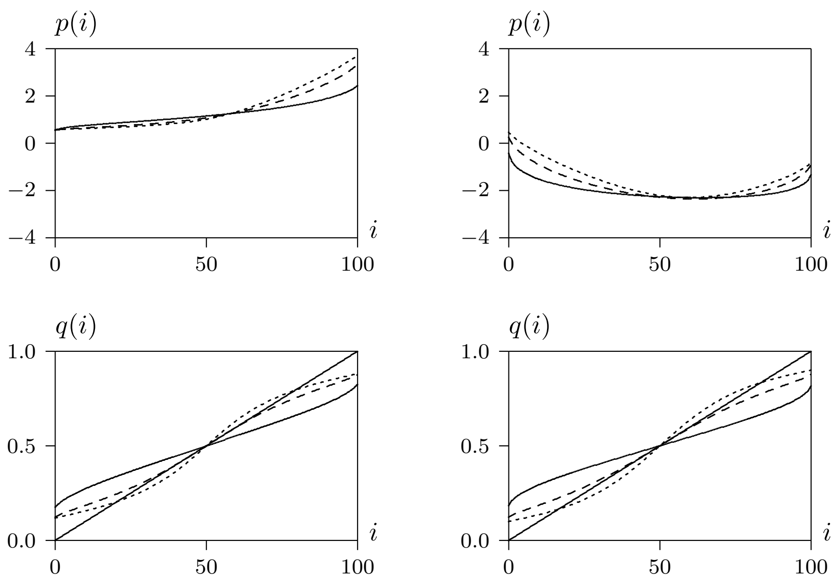


(A) $\lambda(i) = \psi \left(\frac{i}{\eta}\right)^{\nu}$, $\theta(i) = 0$ (B) $\lambda(i) = 0$, $\theta(i) = \psi\, i \left(\frac{i}{\eta}\right)^{\nu}$

Figure 4: Non-linear benefit function. Values of ν: .5, 1, 2 (solid, dashed and dotted line, respectively). In all cases, $\delta = .9, \psi = .3$.

els show the equilibrium price function. These panels essentially confirm the patterns previously observed: if aftermarket benefits accrue to consumers, then the pricing function is increasing; if aftermarket benefits accrue to firms, however, then the pricing function is decreasing for low values of i and increasing for high values of i. The bottom panels show birth rates. As before, $\alpha = 1$ leads to steeper $q(i)$ mappings. Moreover, we see that an increase in the plateau threshold ν also leads to steeper $q(i)$ mappings. To understand the intuition, consider for example the case when $\nu = \frac{1}{4}\,\eta$. This implies that, whenever $\frac{1}{4}\,\eta < i < \frac{3}{4}\,\eta$, *both* firms have maximized their network benefits. As a result, consumers treat both firms equally and the birth probability is not very different from $\frac{1}{2}$. If however $\nu = \frac{3}{4}\,\eta$, for example, then we are closer to the base case considered before.

Finally, Figure 5 shows the results for the case of an S-shaped plateau benefit function. The results are fairly similar to those obtained for other functional forms.



(A) $\lambda(i) = \psi \left(\frac{i}{\nu}\right)^2 \left(3 - 2\,\frac{i}{\nu}\right)$
$\theta(i) = 0$

(B) $\lambda(i) = 0$
$\theta(i) = \psi\, i \left(\frac{i}{\nu}\right)^2 \left(3 - 2\,\frac{i}{\nu}\right)$

Figure 5: S-shaped plateau benefit function. Values of ν: 25, 50, 75 (solid, dashed and dotted line, respectively). In all cases, $\delta = .9, \psi = .5$.

References

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