REPEATED GAMES
Overview

- Context: players (e.g., firms) interact with each other on an ongoing basis
- Concepts: repeated games, grim strategies
- Economic principle: repetition helps enforcing otherwise unenforceable agreements
Repeated games

- Repeated game $\Gamma_T$: Normal-form game $\Gamma$ repeated $T$ times
- $\Gamma$ (a “matrix” game) is called stage game (or one-shot game)
- Strategy in $\Gamma$: choice of row or column
- Strategy in repeated game $\Gamma_T$: a contingency plan indicating choice at time $t$ conditional on history $h_t$
Prisoner’s dilemma with $T = 1$

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<th>Player 2</th>
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<tbody>
<tr>
<td><strong>A</strong></td>
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<td>A</td>
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<td><strong>B</strong></td>
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<td>A</td>
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- B is dominant strategy: unique NE
Prisoner’s dilemma with $T = 2$

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• Repetition of NE of $\Gamma$ constitutes equilibrium of $\Gamma_2$

• Theorem: if $\hat{x}$ is NE of $\Gamma$, then repetition of $\hat{x}$ at every period (ignoring history) is NE of $\Gamma_T$

• Are there additional equilibria?
Grim strategy in PD with $T = 2$

- $t = 1$: choose A
- $t = 2$:
  - If (A,A) was chosen at $t = 1$, then A
  - Otherwise, B
- Check it’s a NE:
  - $t = 1$: deviation earns extra $6 - 5$ but costs $5 - 1$ next period
  - $t = 2$: regardless of history, any rational players picks B
- Therefore, above contingent strategy cannot be an equilibrium
Infinitely repeated prisoner’s dilemma

- Note: indefinitely vs infinitely
- Are there equilibria in $\Gamma_\infty$ other than (B,B) every period?
- Discounted payoff: $\pi_1 + \delta \pi_2 + \delta^2 \pi_3 + \ldots$
  where $\pi_t$ is payoff at time $t$
- Proposed equilibrium strategies:
  - Choose $A$ if $h = \{(A, A), (A, A), \ldots\}$
  - Choose $B$ otherwise
Grim strategy equilibrium

• Equilibrium payoff

\[ \hat{\Pi} = 5 + \delta \ 5 + \delta^2 \ 5 + \ldots = \frac{5}{1 - \delta} \]

• Deviation payoff

\[ \Pi' = 6 + \delta \ 1 + \delta^2 \ 1 + \ldots = 6 + \frac{\delta}{1 - \delta} \]

• \( \hat{\Pi} \geq \Pi' \iff \delta \geq \frac{1}{5} \)

• If \( \delta \) is high enough (future important), deviation does not pay.
Self-enforcing agreements

- Repeated games as foundation for self-enforcing agreements
- Not knowing when game ends (indefinitely repeated) players have something to lose from deviating from “good” action profile
- Most economic relations based on informal contracts
- International agreements (e.g. WTO, Kyoto, etc)
- Positive theories of culture and values
- Agreements are self-enforcing if they form a Nash equilibrium of a repeated “relationship” (game)
Renegotiation

• Suppose that a player chooses $B$ at time $t$
• According to the equilibrium strategies, play reverts to $B$ forever (payoff of 1)
• What stops players from saying “let bygones be bygones” and return to the initial equilibrium?
• But then what stops players from deviating to $B$ in the first place?
• In other words, how credible (renegotiation proof) is the equilibrium system of rewards and punishments?
Example: $T = 1$

Two (Pareto ordered*) Nash Equilibria: (M,C) and (B,R)

* Pareto ordered: both players prefer (M,C) to (B,R).
Example: \( T = 2 \)

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- Repetition of NE of \( \Gamma \) constitutes equilibrium of \( \Gamma_2 \)
- Ignoring history is always a NE of repeated game. Are there additional equilibria?
Grim strategy

- $t = 1$: choose (T,L)
- $t = 2$:
  - If (T,L) was chosen at $t = 1$, then (M,C)
  - Otherwise, (B,R)
- Equilibrium payoff for each player: $5 + 4 > 4 + 4$
- Check it’s a NE:
  - $t = 2$: both (M,C) and (B,R) are NE of one-shot game.
  - $t = 1$: deviation earns extra $6 - 5$ but costs $4 - 1$ next period
Repeated games in the lab

• Stage game:
  – Nature generates potential payoff for players 1 and 2
  – Sum is positive, but one is negative (e.g., 8, −3)
  – Players simultaneously decide whether to accept; if either player rejects, both get zero

• Indefinite repetition of game shows players exchange “favors” frequently. Why?
  – Altruism
  – Intrinsic (backward-looking) reciprocity
  – Instrumental (forward-looking) reciprocity