STRATEGY AND GAMES

Overview

- Context: You're in an industry with a small number of competitors. You're concerned that if you cut your price, your competitors will, too. How do you act? Ditto pretty much any strategic decision: capacity, entry and exit, product positioning.
- Concepts: players, strategies, dominant and dominated strategies, best responses, Nash equilibrium.
- Economic principle: must anticipate others' actions and that your actions might affect theirs.

The field of strategy

- Organizational structure and processes required to implement the firm's plan
- Boundaries of the firm: scale, scope, extent of outsourcing
- Formal analysis of strategic behavior: game theory
- Corporate strategy and business strategy

Game theory

- Formal analysis of strategic behaviour: relations between inter-dependent agents
- Informally, game theory reminds us to:
  - **Understand our competitors:** Our results depend not only on our own decisions but on our competitors' decisions as well
  - **Look into the future:** Decisions taken today may have an impact in future decisions, both by ourselves and by our competitors
  - **Pay attention to information:** Who knows what can make a difference
  - **Look for win-win opportunities:** Some situations are competitive, but others offer benefits to all

Goals

- Our goal is to create an awareness of strategic considerations in many circumstances of business life (and, in fact, of everyday life)
- Our focus is on the play some common games: pricing, capacity, entry and exit, product positioning
- In practice, many of the benefits come from choosing the right games and avoiding the wrong ones. Example: when to avoid cutting prices to gain market share

Historical notes

- John von Neuman was one of the precursors of game theory (and many other things)
- John Nash, of a *Beautiful Mind* fame, was one of the first game theorists to receive the Nobel prize
- The 1995 US spectrum auction was partly designed by game theorist Paul Milgrom
- Game theory is now commonly used by various consulting companies such as McKinsey

What they say

When government auctioneers need worldly advice, where can they turn? To mathematical economists, of course . . . As for the firms . . . their best bet is to go out . . . and hire themselves a good game theorist.

— *The Economist*

Game theory, long an intellectual pastime, came into its own as a business tool.

— *Forbes*

Game theory is hot. — *The Wall Street Journal*

What they say

I think it is instructive to use game theory analysis . . . Game theory forces you to see a business situation over many periods from two perspectives: yours and your competitor's.

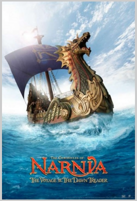
— Judy Lewent, CFO, Merck

At their worst, game theorists represent a throw back to the days of such whiz kids as Robert McNamara . . . who thought that rigorous analytical skills were the key to success. Managers have much to learn from game theory — provided they use it to clarify their thinking, not as a substitute for business experience.

— *The Economist*

Movie release game

- In 2010, Warner Bros. and Fox must decide when to release *Harry Potter* and *The Chronicles of Narnia*
- Two possibilities: November or December
- December is a better month, but simultaneous release is bad for both



Harry Potter and the Deathly Hallows: Part I (released November 19, 2011)
The Chronicles of Narnia: The Voyage of the Dawn Treader (released December 10, 2011)

Game theory: concepts

- What are the elements of a game?
  - Players (in previous example: Warner and Fox)
  - Rules (simultaneously choose release date)
  - Strategies (November, December)
  - Payoffs (revenues)
- What can I do with it?
  - Determine how good each of my strategies is
  - Figure out what my rival is probably going to do

Movie release game

- Suppose total potential revenues (in $ millions) are
  500 in November, 800 in December
- Revenues are split if more than one blockbuster in month

		Fox	
		November	December
Warner	November	250 250	800 500
	December	500 800	400 400

- Coming later: how to analyze game

Class simulation

- You will be paired with a classmate. Identities will not be revealed
- You must choose A or B. Your payoff depends on your choice as well as the other player

		The other	
		A	B
You	A	5	0
	B	6	1

- Please record your choice

How to represent a game

- Matrix form (a.k.a. normal form)
  - Best suited for games with simultaneous decisions
  - Start by looking at dominant, dominated strategies
  - If that fails, look for equilibrium given by intersection of best-response mappings
- Game-tree form (a.k.a. extensive form)
  - Best suited for games with sequential moves
  - Solve game backwards, starting from endnodes
  - Strategies: set of contingent decisions at each node

Solving matrix games

- Given a game (in normal form) how can we analyze it?
- What do we expect rational players to choose?
- What advice would one give to a given player?

Class simulation: prisoner's dilemma

		The other	
		A	B
You	A	5 5	6 0
	B	0 6	1 1

- Dominant strategy: B
- Payoffs (1,1) much worse than (5,5)
- Conflict between individual incentives and joint incentives
- Typical of many business situations

Dominant and dominated strategies

- Dominant strategy: payoff is greater than any other strategy regardless of rival's choice
  - Rule 1: if there is one, choose it
- Dominated strategy: payoff is lower than some other strategy regardless of rival's choice
  - Rule 2: do not choose dominated strategies

Elimination of “dominated” strategies

		Player 2		
		L	C	R
Player 1	T	2 1	0 1	1 1
	M	0 0	3 0	0 0
	B	0 2	1 -2	2 2

Elimination of “dominated” strategies

			Player 2	
		L	C	R
	T	2 1	0 1	1 1
Player 1	M	0 0	3 0	0 0
	B	0 2	1 -2	2 2

Elimination of “dominated” strategies

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Elimination of “dominated” strategies

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Elimination of “dominated” strategies

Player 2

		L	C	R
	T	2 1	0 1	1 1
Player 1	M	0 0	3 0	0 0
	B	0 2	1 -2	2 2

Elimination of “dominated” strategies

1. Player 1 is rational
2. Player 2 is rational and believes Player 1 is rational
3. Player 1 is rational and believes that Player 2 is rational and that Player 2 believes Player 1 is rational
4. Player 2 is rational and believes that Player 1 is rational and that Player 1 believes that Player 2 is rational and that Player 2 believes Player 1 is rational
5. Player 1 is rational and believes that Player 2 is rational and that Player 2 believes that Player 1 is rational and that Player 1 believes that Player 2 is rational and that Player 2 believes Player 1 is rational
6. (You get the drift)

Class simulation

- Choose a number between 0 and 100 (inclusive)
- Let μ be the mean of all players' choices
- Winner: player whose choice is closest to $\mu/2$
- Please write down your number

Dubious application of dominated strategies

		Player 2	
		L	R
Player 1	T	0 1	1 1
	B	0 -1000	1 2

Outcomes of games

- Sometimes a game can be “solved” just by looking at dominant and dominated strategies (e.g., examples above)
- However, there are many games for which this isn’t enough to produce an outcome
- **Nash equilibrium:** Combination of moves in which no player would want to change her strategy unilaterally. Each chooses its best strategy given what the others are doing (or given the beliefs of what others are doing).

Game with no dominant, dominated strategies

			Player 2	
		L	C	R
	T	1 2	2 0	3 0
Player 1	M	1 1	1 1	0 1
	B	1 0	0 2	2 2

Finding Nash equilibria

- A Nash equilibrium is a set of strategies, one strategy for each player, such that: each player, given the strategies of everyone else, is doing the best he or she can
- How do we find this? First, derive **best-response mappings.** For each strategy by player B, find player A's optimal choice. Taken together, these form player A's best-response mapping
- Nash equilibrium: intersection of best-response mappings, i.e., pair of strategy choices (s_A, s_B) such that s_A is optimal given s_B and s_B is optimal given s_A

Best responses

Player 1's best response	
Player 2's strategy	Player 1's best response
L	*T*
C	*B*
R	*B*

Player 2's best response	
Player 1's strategy	Player 2's best response
T	*R*
M	$\{L,C\}$
B	*R*

		Player 2		
		L	C	R
	T	1 2	2 0	3 0
Player 1	M	1 1	1 1	0 1
	B	1 0	0 2	2 2

Best responses and Nash equilibrium

		Player 2		
		L	C	R
Player 1	T	1 $\underline{2}$	2 0	$\underline{3}$ 0
	M	$\underline{1}$ 1	$\underline{1}$ 1	0 1
	B	1 0	0 $\underline{2}$	$\underline{2}$ $\underline{2}$

Nash equilibrium as rest point

- Suppose that, at each stage, either Player 1 or Player 2 chooses best response to what other player was previously playing
- Will this ever stop? If yes, it will stop at a Nash equilibrium
- Example: previous game. Start at (M,R) with Player 2 moving first. Sequence of choices would be:

  (M,R)⟶(M,L)⟶(T,L)⟶(T,R)⟶(B,R)

Notes

- Each player attempts to maximize his or her payoff, not the difference with respect to rival; if rival's payoff is very important (e.g., inducing exit), then this should be taken into account directly
- What do best-response mappings look like when there are dominant or dominated strategies?
- The meaning of simultaneous vs. sequential moves
- Nash's theorem: for any game, there exists at least one (Nash) equilibrium; however, this may involve randomization (mixed strategies)
- Nash equilibrium assumes a lot about what people know (read Adam Brandenburger's letter to the editor of *Scientific American*)

Movie release game (reprise)

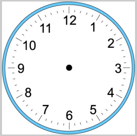
- What is the Nash equilibrium of this game?
- What did actually happen?

		Fox	
		November	December
Warner	November	250 250	800 500
	December	500 800	400 400

Warner (Harry Potter): November 19; Fox (Narnia): December 10.

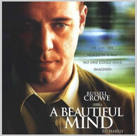
Multiple equilibria and focal points

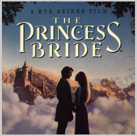
- Schelling experiment (variant):
  - You are to meet X tomorrow in Manhattan
  - Must choose time and place
  - X has been given same instructions as you
  - No communication between you and X
  - If both choose the same time and place, both get \$100; otherwise, both get 0
- What are the Nash equilibria of this game?
- What happens when game is played?

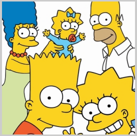


Game theory goes to Hollywood

- Watch video clip from *A Beautiful Mind*.
  Did Ron Howard understand the concept of Nash equilibrium?
- Watch video clip from *The Princess Bride*.
  Does Vizini know anything about game theory?
- Watch video clip from *The Simpsons*.
  Formalize the game played between Bart and Lisa.
  Can you find the Nash equilibrium of this game?







Split or steal

- Read nyusterneconomics blog on “split or steal” (follow the video clips links)
- Formulate the game played at the end of the show
- What is the game’s Nash equilibrium?
- How do you explain the observed behavior?

Rock, Paper and Scissors

Lisa: Look, there's only one way to settle this. Rock-paper-scissors.

Lisa's brain: Poor predictable Bart. Always takes 'rock'.

Bart's brain: Good ol' 'rock'. Nuthin' beats that!

Bart: Rock!

Lisa: Paper.

Bart: D'oh!

Takeaways

- Game theory is a formal approach to strategy
- Highlights impact of strategic interactions among firms or other "players"
- Forces you to consider your competitors' choices
- More coming . . .