PRICING

## Overview

- Context: Many firms face a tradeoff between price and quantity. To sell more, they must charge less. What price should they set? Should they simply apply a standard markup to cost?
- Concepts: demand elasticity, marginal revenue, marginal cost, elasticity rule, market power.
- Bottom line: optimal price is a trade-off between margin and quantity sold, as given by the elasticity rune:

$$
p=\frac{M C}{1+\frac{1}{\epsilon}}
$$

## Example: Ice-cream pricing



## Ice-cream pricing

- Ice-cream truck: driver/operator rents truck, buys ice-cream rom factory, keeps all of the profits
- Fixed cost (truck rental): \$15/hour
- Marginal cost (wholesale cost of ice-cream): \$3
- inverse demand (per hour): $p=10-0.5 q$ (see table on next page)
- What price generates the most profit?


## Ice-cream pricing

| price | demand | revenue | total <br> cost | increm. <br> revenue | increm. <br> cost | profit |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 10.0 | 0.0 | 0.0 | 15.0 |  |  | -15.0 |
| 9.5 | 1.0 | 9.5 | 18.0 | 9.5 | 3.0 | -8.5 |
| 9.0 | 2.0 | 18.0 | 21.0 | 8.5 | 3.0 | -3.0 |
| 8.5 | 3.0 | 25.5 | 24.0 | 7.5 | 3.0 | 1.5 |
| 8.0 | 4.0 | 32.0 | 27.0 | 6.5 | 3.0 | 5.0 |
| 7.5 | 5.0 | 37.5 | 30.0 | 5.5 | 3.0 | 7.5 |
| 7.0 | 6.0 | 42.0 | 33.0 | 4.5 | 3.0 | 9.0 |
| 6.5 | 7.0 | 45.5 | 36.0 | 3.5 | 3.0 | 9.5 |
| 6.0 | 8.0 | 48.0 | 39.0 | 2.5 | 3.0 | 9.0 |
| 5.5 | 9.0 | 49.5 | 42.0 | 1.5 | 3.0 | 7.5 |
| 5.0 | 10.0 | 50.0 | 45.0 | 0.5 | 3.0 | 5.0 |
| 4.5 | 11.0 | 49.5 | 48.0 | -0.5 | 3.0 | 1.5 |

## Optimal pricing: calculus

- Since there is a one-to-one correspondence between price and demand (the demand curve), we can either determine optimal price or optimal output
- Profit is normally an inverted-U-shaped function of output
- If slope is positive, then higher output lads to higher profit
- If slope is negative, then lower output leads to higher profit
- At the optimal output level, derivative of profit with respect to output is zero. This is a necessary (though not sufficient) condition


## Profit maximization

$\pi(q)$


## Profit maximization: calculus

- Profit and marginal profit:

$$
\begin{aligned}
\pi(q) & \equiv R(q)-C(q) \\
\frac{d \pi(q)}{d q} & =\frac{d R(q)}{d q}-\frac{d C(q)}{d q}
\end{aligned}
$$

- Marginal revenue: $M R \equiv \frac{d R(q)}{d q}$
- Marginal cost: $M C \equiv \frac{d C(q)}{d q}$
- Profit maximization implies that $\frac{d \pi(q)}{d q}=0$, which is equivalent to

$$
M R=M C
$$



## Notes on marginal revenue

- What do you get from selling an extra unit?

You get the price for which you sell it, but the additional (marginal) revenue is less than that.

- Price must be lowered in order for an extra unit to be sold; this lowers the marginal on all units sold.
- Formally,

$$
M R \equiv \frac{d R}{d q}=\frac{d(p \times q)}{d q}=p+\frac{d p}{d q} q<p
$$

## The elasticity rule

$$
M R=p+\frac{d p}{d q} q=p+\frac{d p}{d q} \frac{q}{p} p=p+\frac{1}{\frac{d q}{d p} \frac{p}{q}} p=p\left(1+\frac{1}{\epsilon}\right)
$$

Therefore, $M R=M C$ implies that $p\left(1+\frac{1}{\epsilon}\right)=M C$, or

$$
p=\frac{M C}{1+\frac{1}{\epsilon}}
$$

Alternatively, this may be written as $p-M C=-p \frac{1}{\epsilon}$, or simply

$$
m \equiv \frac{p-M C}{p}=\frac{1}{-\epsilon}
$$

## Demand elasticity and monopoly margin



## Demand elasticity and monopoly margin



## Margin and markup

- Two alternative ways of measuring gap between price and marginal cost:

$$
\begin{aligned}
m & \equiv \frac{p-M C}{p} \\
k & \equiv \frac{p-M C}{M C}
\end{aligned}
$$

- Corresponding elasticity rules:

$$
\begin{aligned}
m & =\frac{1}{-\epsilon} \\
k & =\frac{1}{-\epsilon-1}
\end{aligned}
$$

## Example

- Product: new drug, protected by patent
- Estimated elasticity: -1.5 (constant)
- Marginal cost: $\$ 10$ (for a 12-dose package)
- What's the profit maximizing price?
- What are values of margin, markup at optimal price?
- Check elasticity rules


## Ice-cream pricing (reprise)

- Recall that $F=15, M C=3, p=10-0.5 q$
- Elasticity is not constant, so elasticity rule is not very useful
- Apply $d \pi(q) / d q=0$ directly (or $M R=M C$ ):

$$
\begin{aligned}
& \pi(q)=\left(10-\frac{1}{2} q\right) q-3 q-15 \\
& \frac{d \pi}{d q}=-\frac{1}{2} q+\left(10-\frac{1}{2} q\right)-3 \\
& \frac{d \pi}{d q}=0 \Rightarrow q=7 \\
& \Rightarrow p=10-\frac{1}{2} q=6.5
\end{aligned}
$$

## Ice-cream pricing (reprise)

- We didn't use the elasticity rule to find $p^{*}$, but nevertheless elasticity rule holds at $p=p^{*}$

$$
\begin{aligned}
& \frac{1}{-\epsilon}=-\frac{d p}{d q} \frac{q}{p}=\frac{1}{2} \frac{7}{6.5}=.5385 \\
& m=\frac{p-M C}{p}=\frac{6.5-3}{6.5}=.5385
\end{aligned}
$$

## Optimal pricing: graphical derivation



## Optimal pricing: graphical intuition



## Comments on elasticity rule

- Standard markup is a bad idea: you want higher markups for products with lower elasticities
- If $|\epsilon|<1$, always better off by increasing price
- Every firm is a "monopolist," but the extent of its monopoly power is given by $1 /|\epsilon|$
- Question: "what will the market bear?" Answer: MC / $\left(1+\frac{1}{\epsilon}\right)$
- If a firm sells multiple products, some complications may arise. More on this below


## Complications, I: demand interactions

- What if firm sells two products that are related?
- Examples:
- Substitutes (e.g., Unilever)
- Complements (e.g., Gillette)
- Bundles (e.g., supermarkets)
- How does this influence optimal pricing strategy?


## Complications, II: dynamic interactions

- What if firm sells a product over a number of periods?
- Examples:
- Buz effects (e.g., movies)
- Network effects (e.g. social networks)
- Habituation effects (e.g., videogames, cigarettes)
- How does this influence optimal pricing strategy?


## Takeaways

- Optimal price depends on:
- marginal cost
- what the market will bear (demand elasticity)
- In a competitive market (high $|\epsilon|$ ), optimal markup is low. If your product has unique characteristics and/or you're the only producer (low $|\epsilon|$ ), then optimal markup can be high.
- If you sell various related products, then optimal pricing becomes more complicated
- What's missing:
- more complex pricing schemes (Chapter 6)
- competition (Chapters 8 and 9)

