Notes on Logarithms

■ **Why study logarithms?** Logarithms turn out to be useful in lots of places: elasticities, compound interest, growth rates, and so on. In Introduction to Industrial Organization, we'll be using logarithms to estimate and apply demand elasticities; but logarithms will pop up in other contexts as well.

■ **What are logarithms?** The natural logarithm of a number x comes from the power of a number e, which is approximately equal to 2.718. If $x = e^y$, then y is the logarithm of x. We write $y = \ln x$ or $y = \log x$. There are other logarithms based on powers of other numbers (you can use any positive number you like, not just e), but we'll stick with e. If you're using Excel, the natural logarithm of x is written "ln(x)."

Suppose, instead, you know the logarithm y of x. How do you find x? From the definition, $x = e^y$. In Excel, this is written "exp(y)."

■ **How do they work?** The most useful properties of logarithms are

$$\begin{aligned}\log(x\,y) &= \log x + \log y \\ \log \frac{x}{y} &= \log x - \log y \\ \log(x^y) &= y\,\log x\end{aligned}$$

In addition, the derivative of the natural log of x is the inverse of x:

$$\frac{d\,\log x}{d\,x} = \frac{1}{x}$$

■ **Mathematical example.** Find the value x such that

$$\frac{\log 5.63 - \log x}{\log 11 - \log 13} = -1.20$$

Answer:

$$\begin{aligned}\log x &= \log 5.63 + 1.20\,(\log 11 - \log 13) \\ &= 1.53\end{aligned}$$

Hence,

$$x = \exp(1.53) = 4.61$$

■ **Can you give us an example of an application?** Sure. In Chapter 2 you will find an application of logarithms to estimating the price elasticity of demand. Another application is compound growth. GDP per capita in Korea was \$1000 in 1960, \$6000 in 1990. (These are rough estimates, measured in 1990 US dollars — i.e., corrected for inflation). What was the average annual growth rate?

We're looking for a number g satisfying

$$6000 = (1+g)^{30}\, 1000$$

How do we find g? One way is to use logarithms. Note that

$$\log\left(\frac{6000}{1000}\right) = 30\, \log(1+g)$$

Since $\log 6 = 1.792$, $\log(1+g) = \frac{1.792}{30} = 0.0597$, and $1+g = \exp(0.0597) = 1.062$. The growth rate was 6.2% a year, which is extraordinarily high.

Similar calculations to these show up in present value calculations.