10.1. Market size and market structure. Explain in words why the number of firms in a free-entry equilibrium may be less than proportional to market size.

Answer: The explanation lies in the fact that as the number of firms increases, so does competition. As a result, prices will fall, reducing the margin, \( p - c \). Therefore, variable profit per unit of market size decreases, making the number of firms the market can sustain increase less than proportionally to market size.

10.2. Single market. Suppose that two countries, initially in autarchy, decide to create a single market. For simplicity, assume that, in both economies, there is only one product. Demand for this product is given by \( D_i = S_i (a - p_i) \), \( i = 1, 2 \), where \( S_i \) is a measure of country \( i \)'s size. Upon the creation of a single market, total demand is given by the horizontal sum of the two initial demands.

Assuming there is free entry and that firms compete a la Cournot, determine the equilibrium number of firms in autarchy and after the completion of the single market. Interpret the results.

Answer: In autarchy we have \( p_i = a - D_i / S_i \). Assuming that the cost function takes the form \( C(q_{ik}) = F + c q_{ik} \) the equilibrium number of firms in market \( i \) is given by (10.1), that is,

\[
\Pi(n_i) = S_i \left( \frac{a - c}{n_i + 1} \right)^2 - F
\]

The equilibrium number of entrants is given by (10.3), that is,

\[
\hat{n}_i = \left[ (a - c) \sqrt{\frac{S_i}{F}} - 1 \right]
\]

After the completion of the single market the size of the market increases and demand becomes

\[
D_{1+2} = D_1 + D_2 = (S_1 + S_2)(a - p)\]
We now have
\[
\hat{n}_{1+2} = \left( a - c \right) \sqrt{\frac{S_1 + S_2}{F}} - 1
\]

Note that, if the values of \( n \) are large, then
\[
\hat{n}_1 + \hat{n}_2 \approx (a - c) \sqrt{\frac{S_1}{F}} + (a - c) \sqrt{\frac{S_2}{F}}
\]
\[
= \frac{a - c}{\sqrt{F}} \left( \sqrt{S_1} + \sqrt{S_2} \right)
\]
\[
> \frac{a - c}{\sqrt{F}} \left( \sqrt{S_1 + S_2} \right)
\]
\[
\approx \hat{n}_{1+2}
\]

In words, \( n_{1+2} \) is lower than \( n_1 + n_2 \): the free trade agreement implies that some firms will exit. The explanation for this effect is the same as in Exercise 10.1.

10.3. California and Montana. Adapted from an exercise written by T. Bresnahan.20

The number of imported automobiles in California is four times higher than in Montana, in per capita terms. The population of Californian is mainly urban, whereas the population of Montana is mainly rural. How do demographic differences and the model presented in Section 10.1 explain the differences in consumption patterns?

Answer: One possible explanation is related to the answers to Exercises 10.1 and 10.2. As Exercise 10.2 shows, when two markets get merged, the total number of firms increases (though by less than simply adding the number of firms). A larger number of firms implies a lower price. This in turn implies that quantity demanded is higher.

Comparing two states with different degrees of urbanization is akin — to a certain extent — to analyzing the effects of a single market: an urbanized state is like a state with rural population where many small markets were turned into a single, large market. In this sense, one would expect urban states to have more competitive markets, thus with lower prices and a higher consumption on a per capita basis.

In other words, theory predicts that smaller markets will have fewer firms and higher margins. The fact that the population of Montana is mainly rural implies that the typical market for a car dealer is smaller than in California.

10.4. Market size and market structure. In some industries, the number of firms increases as market size increases. In other industries, the number of firms seems remarkably stable despite changes in market size. Discuss.

Answer: Refer to the earlier discussion on the difference between endogenous and exogenous entry costs.

10.5. Retail in Switzerland. Retail in Switzerland is mostly dominated by highly profitable cartels. The Swiss authorities anticipate the gradual collapse of these cartels as the country becomes better integrated with the rest of Europe. OECD, by contrast, hold a more sceptical view, claiming that the collapse of cartels does not necessarily lead to more
competitive markets; rather, they add, cartel breakdowns are frequently associated with an increase in concentration. Which prediction seems more reasonable? Are the two views inconsistent?

Answer: Integration is likely to imply greater competition from foreign suppliers. Lower margins will then imply that the Swiss market cannot hold the same number of firms as currently. It is therefore possible that the two predictions hold true: that prices go down and that the industry becomes more concentrated.

10.6. Market definition and market structure. Consider the following goods: cement, mineral water, automobiles, retail banking. In each case, determine the relevant market boundaries and present an estimate of the degree of concentration.

10.7. Cost reduction and the Herfindahl and Lerner Indexes. Consider an industry where demand has constant price elasticity and firms compete in output levels. In an initial equilibrium, both firms have the same marginal cost, \( c \). Then Firm 1, by investing heavily in R&D, manages to reduce its marginal cost to \( c' < c \); a new equilibrium takes place.

(a) What impact does the innovation have on the values of \( H \) and \( L \)?

Answer: Initially, \( H = \frac{1}{2} \). In the new equilibrium, we have a duopoly with different marginal costs, which implies \( H > \frac{1}{2} \). Specifically, let \( s \) be the largest firm market shares. Then

\[
H = s^2 + (1 - s)^2 = 1 - 2s(1 - s)
\]

This is increasing in \( s \), attaining a maximum when \( s = 1 \) (monopoly) and a minimum when \( s = \frac{1}{2} \). Since \( L = H/(\epsilon) \) and \( \epsilon \) is constant, it follows that \( L \) increases as a result of the innovation.

(b) What impact does the innovation have on consumer welfare?

Answer: From our treatment of the Cournot model, we know that a decrease in one of the firms’ cost leads to a lower price, thus higher consumer surplus.

(c) What do the previous answers have to say about \( L \) as a performance measure?

Answer: \( L \) may not be a good indicator of consumer welfare when the comparative statics corresponds to changes in cost levels. By contrast, if cost levels remain constant and there is only variation in the number of firms or the price elasticity of demand, then \( L \) provides a good indicator of how consumer welfare varies.

10.8. Barriers to entry and welfare. “Barriers to entry may be welfare improving.” What particular industry characteristics might make this statement valid?

Answer: Following the discussion in section 10.4, we said that free entry is decreasing welfare when the business stealing effect dominates. For this to happen, as in the example with retail banking, “products” should be homogenous (product differentiation is unimportant) and competition should be soft. In this case, paying a fee for setting up a branch represents a barrier to entry and may act as an efficient means of blocking excessive entry.
10.9. Number of competitors and equilibrium profits. Derive Equation (10.1).

**Answer:** Let the inverse demand curve be given by

\[ P = a - Q/S \]

\( S \) is a measure of market size. If the only difference between market \( A \) and market \( B \) is that the latter is twice the size of the first, then the demand curve in market \( B \) is as in \( A \) but with a value of \( S \) that is twice that of market \( A \)’s.

Each firm’s profit, in case it enters the market, is given by

\[ \Pi = Pq_i - F - cq_i = \left(a - Q/S - c\right)q_i - F \]

The first-order condition for profit maximization is

\[ a - Q/S - c - q_i/S = 0 \]

In a symmetric equilibrium, we have \( q_i = q = Q/n \), where \( n \) is the number of active firms. It follows that

\[ a - nq/S - q/S = 0 \]

or

\[ q = \frac{a - c}{n + 1} \]

Equilibrium price is given by

\[ p = a - nq/S = a - n \frac{a - c}{n + 1} \]

Substituting in the profit function, we get

\[ \Pi(n) = (p - c)q - F = \left(a - n \frac{a - c}{n + 1} - c\right)S \frac{a - c}{n + 1} - F \]

\[ = \frac{a - c}{n + 1} S \frac{a - c}{n + 1} - F \]

\[ = S \left(\frac{a - c}{n + 1}\right)^2 - F \]

10.10. Market structure and market power under Cournot competition. Derive equation (10.6).

**Answer:** Firm \( i \)’s profit is given by

\[ \pi_i = pq_i - C_i \]
where \( p = P(Q) \), \( P(\cdot) \) is the inverse demand curve, and \( Q \) is total output. The first-order condition for profit maximization is given by

\[
\frac{dp}{dq_i} q_i + p - MC_i = 0
\]

Since \( Q = \sum_i q_i \), \( \partial p / \partial q_i = \partial p / \partial Q \). We can thus rewrite the above equation as

\[
p - MC_i = -\frac{dp}{dQ} q_i
\]

The industry Lerner index is defined as

\[
L = \sum_{i=1}^{n} s_i \frac{p - MC_i}{p}
\]

Since \( s_i \equiv q_i/Q \) and \( p - MC_i = -\frac{dp}{dQ} q_i \), this can be re-written as

\[
L = \sum_{i=1}^{n} \frac{q_i}{Q} \frac{dp}{dQ} q_i = -\frac{dp}{dQ} \frac{Q}{p} \sum_{i=1}^{n} s_i^2 = -\frac{1}{\frac{dQ}{dp} \frac{Q}{p}} \sum_{i=1}^{n} s_i^2 = \frac{H}{-\epsilon}
\]

**10.11. Scale economies.** Show that the coefficient of scale economies, \( AC/MC \), is greater than one if and only if average cost is decreasing.

**Answer:** First note that, if \( \rho > 1 \), then \( AC \) is decreasing, that is, \( dAC/dq < 0 \). Also note that

\[
\frac{dAC}{dq} = \frac{d}{dq} \left( \frac{C}{q} \right) = \frac{1}{q} \frac{dC}{dq} - C \frac{1}{q^2} = \frac{1}{q} (MC - AC) < 0
\]

since

\[
\rho \equiv \frac{AC}{MC} > 1
\]

Second, note that, if \( AC \) is decreasing, then \( \rho > 1 \). If \( AC \) is decreasing, we know that \( dAC/dq < 0 \), or \( \frac{1}{q} (MC - AC) < 0 \). From the latter we have that \( MC - AC < 0 \), therefore, \( \rho > 1 \).

**10.12. Technology and market structure.** Consider an industry with market demand \( Q = a - p \) and an infinite number of potential entrants with access to the same technology. Initially, technology is given by \( C = F + cq \). A new technology allows for a lower marginal cost, \( c' < c \), at the expense of a higher fixed cost, \( F' > F \).

(a) What can you say about the effect of the new technology on equilibrium price?

**Answer:** For a given number of firms, a lower marginal cost \( (c' < c) \) implies a lower equilibrium price. However, to the extent that \( F' > F \), it is also possible that the number of firms in a free-entry equilibrium declines, which in turn implies a higher equilibrium price. As a result, one cannot say in general whether equilibrium price increases or decreases.
Specifically, suppose that firms compete a la Cournot and assume that the value of \( n \) is sufficiently large that we can ignore the integer constraint (that is, the fact that the number of firms must be an integer number). In equilibrium, the number of active firms is given by

\[
    n = (a - c) \sqrt{\frac{1}{F}} - 1
\]

For a given \( n \), equilibrium price is given by

\[
    p = \frac{a + nc}{n + 1}
\]

We can therefore define “iso-price” curves by the expression

\[
    p = \frac{a - c + c(a - c)\sqrt{\frac{1}{F}}}{(a - c)\sqrt{\frac{1}{F}}} = c + \sqrt{F}
\]

(b) Suppose that \( a = 10, F = 2, F' = 3, c = 2, c' = 1 \). Determine equilibrium price under each of the two technologies.

**Answer:** Let \( n_1 \) and \( n_2 \) be the equilibrium values of \( n \) under the old and the new technologies, respectively. In general,

\[
    n = \left[ (a - c) \sqrt{\frac{S}{F}} - 1 \right]
\]

It follows that

\[
    n_1 = \left[ (10 - 2) \sqrt{\frac{1}{2}} - 1 \right] = 4
\]

\[
    n_2 = \left[ (10 - 1) \sqrt{\frac{1}{3}} - 1 \right] = 4
\]

It follows that equilibrium price is given by

\[
    p_1 = \frac{a + nc}{n + 1} = \frac{10 + 4 \times 2}{5} = 3.6
\]

\[
    p_2 = \frac{a + nc}{n + 1} = \frac{10 + 4 \times 1}{5} = 2.8
\]

**\( 10.13. \) Alternative production technologies.** Consider an industry with a homogeneous product where firms set output (or capacity) levels and price is determined by total output (or capacity). Suppose there is a large number of potential entrants and that each firm can choose one of two possible technologies, with cost functions \( C_i = F_i + c_i q_i \) \( (i = 1, 2) \).

(a) Derive the conditions for a free-entry equilibrium.

**Answer:** From Exercise 8.8, we know that equilibrium profits in an asymmetric Cournot-like oligopoly are given by

\[
    \pi_i = \frac{1}{b} \left( \frac{a - nc_i + \sum_{j \neq i} c_j}{n + 1} \right)^2 - F_i
\]
Suppose there are two groups of firms: $n_1$ firms choose technology 1 and $n_2$ firms choose technology 2, where $n = n_1 + n_2$. Then

$$\hat{\pi}_i = \frac{1}{b} \left( \frac{a - n c_i + (n_i - 1) c_i + n_j c_j}{n + 1} \right)^2 - F_i$$

where $j \neq i$.

In a free-entry equilibrium, for type 1 and type 2 firms must make positive profits. Moreover, a potential entrant would need to make negative profits were that entrant to choose either of the two available technologies. We thus have four different constraints.

1. $$\frac{1}{b} \left( \frac{a - n c_i + (n_i - 1) c_i + n_j c_j}{n + 1} \right)^2 - F_i \geq 0$$
2. $$\frac{1}{b} \left( \frac{a - (n + 1) c_i + n_i c_i + n_j c_j}{n + 2} \right)^2 - F_i \leq 0$$

(b) Show, by means of numerical example, that there can be more than one equilibrium, with different numbers of large and small firms.

**Answer:** Suppose that $a = 100$, $b = 1$, $F_1 = 15$, $c_1 = 0$, $F_2 = 12$, $c_2 = .5$. The following pairs $(n_1, n_2)$ satisfy the above equations: (5,22); (24,0); (3,24).


Suppose you only know the value of the market shares for the largest $m$ firms in a given industry. While you do not possess sufficient information to compute the Herfindahl index, you can find a lower and an upper bound for its values. How?

**Answer:** A lower bound would result from an industry where, in addition to the top $m$ firms, there is a very large number of firms with a very small market share. In the limit of infinitesimal shares, the value of $H$ would be $H = \sum_{i=1}^{m} s_i^2$. An upper bound would result from an industry where all the remaining firms have the same market share as the $m$-th firm. The value of $H$ would then be $H = \sum_{i=1}^{m} s_i^2 + (1 - \sum_{i=1}^{m} s_i)) s_m$. (Notice that the remaining firms would be $(1 - \sum_{i=1}^{m} s_i)) / s_m$ in number.

The above lower and upper bounds are frequently very close, so a fairly good approximation if often possible.

### 10.15. Product differentiation and market structure.

Consider the monopolistic competition model, presented in Section 4.3. What is, according to this model, the relation between the degree of product differentiation and market structure?

**Answer:** Refer to Chapter 4. The greater the degree of product differentiation, the steeper the demand curve $d$ faced by each firm. In the long run, price equal average cost. Therefore, the steeper $d$ is the lower each firm’s output is in the long run equilibrium. We would therefore expect a more fragmented market structure when the degree of product differentiation is higher.

### 10.16. Doctors and plumbers.

Consider the structure of geographically isolated markets in the US (small towns) in the following businesses: doctors, dentists, plumbers.
It can be shown that the minimum town size that justifies the entry of a second doctor is approximately 3.96 times the required size for the first doctor to enter. For plumbers, the number is 2.12. How can these numbers be interpreted?

**Answer:** The higher number for doctors has two interpretations. The first one is that competition between two doctors is very intense, so that it would take a much larger market before the second doctor could recoup entry costs. The second interpretation is that there are specific barriers to entry by a second doctor which are not present in the case of a plumber.

**10.17. Advertising costs.** Consider the following model of entry into an advertising-intensive industry. To simplify the analysis, and to concentrate on the effects of advertising, suppose that there is no price competition. Specifically, the value of the market, in total sales, is given by $S$. (One can think of a demand curve $D(p)$ and an exogenously given price, whereby $S = pD(p)$.) $S$ is therefore a measure of market size.

Each firm must decide whether or not to enter the industry. Entry cost is given by $F$. If a firm decides to enter, then it must also choose how much to invest in advertising; let $a_i$ be the amount chosen by firm $i$. Finally, firm $i$’s market share, $s_i$, is assumed to be equal to its share of the industry total advertising effort:

$$s_i = \frac{a_i}{\sum_{j=1}^{n} a_j} = \frac{a_i}{A}$$

where $n$ is the number of firms in the industry and $A \equiv \sum_{i=1}^{n} a_i$ is total industry advertising.

(a) Show that each firm $i$’s optimal level of advertising solves $S(A - a_i)/A^2 - 1 = 0$

**Answer:** The profit of each firm is given by

$$\pi_i = s_i S - a_i - F$$

Therefore, each firm is solves

$$\max_{a_i} \frac{a_i}{A} S - a_i - F$$

The first order condition is given by

$$\frac{A - a_i}{A^2} S - 1 = 0$$

Notice that $A = \sum a_i$. Therefore, when deriving the first-order condition we must take into account the effect of changes in $a_i$ on the total $A$.

(b) Show that, in a symmetric equilibrium, $a = S (n - 1)/n^2$, where $a$ is each firm’s level of advertising.

**Answer:** In a symmetric equilibrium we have $a_i = a = A/n$, or alternatively $A = na$. The above result then implies

$$\frac{na - a}{(na)^2} S - 1 = 0$$

Solving with respect to $a$ we obtain the equation in the text.
(c) Show that equilibrium profit is given by $\pi = S/n^2 - F$

**Answer:** As shown above, firm profit is given by

$$\pi_i = s_i S - a_i - F$$

Substituting the equilibrium values, we get

$$\pi = \frac{1}{n} S - \frac{n-1}{n^2} S - F = \frac{S}{n^2} - F$$

(d) Show that the equilibrium number of entrants is given by the highest integer lower than $\sqrt{S/F}$.

**Answer:** The equilibrium requires $\pi(n) \geq 0$ and $\pi(n + 1) \leq 0$, where $\pi(n)$ denotes equilibrium profits when there are $n$ entrants. This implies that the equilibrium value of $n$ is the largest integer such that $\pi(n) \geq 0$. Solving

$$\frac{S}{n^2} - F = 0$$

with respect to $n$ we get

$$n = \sqrt{\frac{S}{F}}$$

Therefore,

$$\hat{n} = \left\lfloor \sqrt{\frac{S}{F}} \right\rfloor$$

(e) Interpret this result in light of the previous discussion on the effects of endogenous entry costs.

**Answer:** In equilibrium, advertising expenditures increase with market size. This is an instance of an endogenous entry cost. This implies that the number of firms increases less than proportionately with respect to market size — even though price is (by assumption) constant.

10.18. Entry and welfare. Consider a homogeneous product industry with inverse demand function $P(Q)$ where every firm has the same cost function: $C(q)$. Suppose that firms decide sequentially whether or not to enter the industry and that the number of firms can be approximated by a continuous variable $n$. Show that, if (a) an increase in $n$ leads to a decrease in $q_n$ (the equilibrium output per firm when there are $n$ firms); and (b) equilibrium price is greater than marginal cost; then the equilibrium number of firms is too high from a social welfare point of view. Hint: derive the condition such that a firm is indifferent between entering and not entering; then show that, at that value of $n$, the derivative of social welfare with respect to $n$ is negative.

**Answer:** The question we are trying to address is the relation between the optimal number of firms, $n^*$, and the equilibrium number of firms, $\hat{n}$. The optimal number of firms maximizes total welfare, which is given by
\[ W(n) \equiv \int_{0}^{n q_n} P(x) \, dx - n \, C(q_n) - n \, F \]  
(10.7)

where, as before, \( n \) is the number of firms and \( q_n \) each firm’s output (given that there are \( n \) active firms).

The effect of additional entry on welfare is given by

\[ W'(n) = P(n q_n) \left( n \frac{\partial q_n}{\partial n} + q_n \right) - C(q_n) - n \, C'(q_n) \frac{\partial q_n}{\partial n} \]

The equilibrium number of firms, \( \hat{n} \), is given by the zero-profit condition

\[ P(\hat{n} q_{\hat{n}}) \, q_{\hat{n}} - C(q_{\hat{n}}) = 0 \]

Substituting this in (??), we get

\[ W'(\hat{n}) = \hat{n} \left( P(\hat{n} q_{\hat{n}}) - C'(q_{\hat{n}}) \right) \frac{\partial q_n}{\partial n} \]

We conclude that, if margins are positive (\( P > C' \)) and there is a business stealing effect \((\partial q_n / \partial n < 0)\), then, at the equilibrium level of entry, further entry would reduce social welfare; and, conversely, less entry would increase welfare. In other words, there is excessive entry in equilibrium.

Notice that, by using differential calculus, this result abstracts from the fact that \( n \) must be an integer value. When this problem is taken into consideration, examples may be found where there is insufficient entry in equilibrium. For example, suppose that marginal cost is constant and that duopoly price is equal to marginal cost (Bertrand competition). In this case, even if the entry cost is very small, the equilibrium number of firms is just one. However, if the entry cost is indeed very small, then society would be better off with a second competitor.

**Applied exercises**

10.19. Industry evolution. Choose an industry for which you can find historical data. Describe the industry’s evolution in terms of number of firms and the firm size distribution. Explain what factors determined such evolution and how they relate to the discussion in the present chapter.

10.20. AirBnB. Write a short essay on the birth and evolution of AirBnB. Focus on the legal difficulties the new service has experienced and compare the normative and capture theories of entry regulation as ways to explain the facts.