Review and practice exercises

9.1. Tacit collusion. Industries A and B can be characterized by a series of parameters: 
- $n$, the number of firms, is 8 in both industries; 
- $r$, the annual interest rate, is 10% in both countries; 
- $f$, the frequency with which firms interact (number of times per year), is 1 in industry A and 12 in industry B; 
- $g$, the industry growth rate, is 10% in industry A and -30% in industry B; finally, 
- $h$, the likelihood that the industry will continue in existence into the next period, is 80% in industry A and 100% in industry B.

In which of the two industries do you think tacit collusion is more likely to take place? Briefly justify your answer.

Answer: The effective discount factor is given by

$$\delta = \frac{h (1 + g)}{1 + r/f}$$

Substituting the above values, we get $\delta_A = 0.8$ and $\delta_B = 0.694$. Based on these calculations, and considering the number of firms is the same in both industries, we would expect tacit collusion to be easier in industry A. Although interaction between firms is more frequent in industry B, the fact that it is a declining industry ultimately renders collusion very difficult: the promise of continuing collusion in the future is of little importance, leading firms to have a greater incentive to cheat on a collusion agreement.

9.2. $n$ firm oligopoly. Consider a price-setting oligopoly with $n$ firms, all with constant marginal cost $c$. Suppose market demand is given by $D(p)$ and the discount factor is 0.8. Determine the maximum number of firms such that there exists an equilibrium with monopoly pricing.

Answer: The no-deviation constraint is given by

$$\frac{1}{n} \left( \frac{\pi^M}{1 - \delta} \right) \geq \pi^M$$

or simply

$$n \leq \frac{1}{1 - \delta} = 5$$
9.3. Repeated interaction. Explain why collusive pricing is difficult in one-period competition and easier when firms interact over a number of periods.

**Answer:** Refer to Section 9.1. In one-period competition, each firm has a strong incentive to deviate from the pre-agreed collusive price, since the gains from deviating are higher than the losses. In terms of the example in Section 9.1, had the duopolists interacted in only one period, the gain would be given by one half of monopoly profits, while the loss from deviating would be 0. We would then be led to the usual Nash-Bertand equilibrium when both firms price at marginal cost.

If, however, firms interact over a number of periods, history, in the form of past pricing behavior, becomes important. Deviation from the collusive price in one period can be met with punishment (deviation) in future periods. Hence, the original defector must weigh short-term gains against long-term losses, made possible exactly by multi-period interaction.

9.4. Airbus and Boeing. Boeing and Airbus seem to cycle between periods of severe price competition and pledges that they will not sink into another war. Based on the analysis of Section 9.1, why do you think it is so difficult for aircraft manufacturers to collude and avoid price wars?

**Answer:** Refer to Section 9.1. Aircraft manufacturers receive orders infrequently. Moreover, the terms of each sale are seldom made public. For these reasons, it is very difficult for them to collude. The incentive to cheat on a tacit or explicit agreement would be very high because: (a) the short run is very important with respect to the long run (low discount factor); (b) the probability that cheating would be detected is low.

9.5. Price wars. “Price wars imply losses for all of the firms involved. The empirical observation of price wars is therefore a proof that firms do not behave rationally.” True or false?

**Answer:** False. As Section 9.2 shows, price wars may be part of the equilibrium of a game played between rational firms.

9.6. Airline fare wars I. Empirical evidence from the US airline industry suggests that fare wars are more likely when carriers have excess capacity, caused by GDP growth falling short of its predicted trend. Fare wars are also more likely during the Spring and Summer quarters, when more discretionary travel takes place.**28** Explain how these two observations are consistent with the theories presented in Section 9.2.

**Answer:** Refer to Section 9.1 and 9.2. The first model in Section 9.2 (secret price cuts) predicts that price wars start in periods of unexpected low demand. This is consistent with the first observation above. However, the effect of unexpected low demand is also consistent with a theory of price wars caused by financial distress (see the end of Section 9.2). The observation that fare wars take place during periods of higher demand is consistent with the second model in Section 9.2 (demand fluctuations).
9.7. Airline fare wars II. A 1998 news article reported that

Delta Air Lines and American Airlines tried to raise leisure air fares 4% in most
domestic markets, but the move failed Monday when lone-holdout Northwest
Airlines refused to match the higher prices.

The aborted price boost illustrates the impact Northwest’s woes already are
having on the industry. Months of labor unrest ... are prompting passengers to
book away from the fourth largest carrier.\textsuperscript{29}

What does this say about the nature of price dynamics in the airline industry?

\textbf{Answer:} The observation seems consistent with the theory that financially distressed firms,
which effectively have a lower discount factor \( \delta \), render collusive agreements more difficult
to sustain.

forest-products companies experienced an improvement in their results. The industry, ana-
lysts said, was in a cyclical upswing: not only was demand increasing at a moderate pace;
more important, the industry practiced restraint in keeping low production levels, thus
providing support for higher prices.\textsuperscript{30}

How do you interpret these events in light of the models presented in Section 9.2?

\textbf{Answer:} The analysis of Section 9.1 predicts that collusion is easier in growing industries
(the promise of future profits under collusion is worth more). This is consistent with the fact
that “restraint in keeping low production levels” took place during the “cyclical upswing.”

9.9. Export cartels. In 1918, the US Congress passed a law allowing American firms
to form export cartels. Empirical evidence suggests that cartels were more likely to be
formed in industries where American exporters had a large market share, in capital-intensive
industries, in industries selling standardized goods, and in industries that enjoyed strong
export growth.\textsuperscript{31} Discuss.

\textbf{Answer:} The effect of export growth seems consistent with the analysis in Section 9.1. The
effect of standardization may correspond to the fact that it is easier to monitor collusion
with a standardized product (however, the effect of product differentiation on collusion is
a controversial issue). The effect of market share is consistent with the analysis in Section
9.3 (concentration facilitates collusion).

9.10. Cartel leniency. Many antitrust authorities throughout the world have implemented
leniency programs targeted at busting secret cartels. These programs offer immunity from
prosecution to firms who blow the whistle on their co-cartel conspirators. These programs
have proven extremely successful: in the US, from 1993 to 2000 the total amount of fines
for anti-competitive behavior increased by twentyfold.

Show how the leniency programs first implemented by the US Department of Justice
and later replicated in many countries changes the rules of the game played between firms
in a secret cartel.
Consider an industry with two firms who secretly have been colluding to fix prices. Absent a leniency program, their dominant strategy is not to reveal any information to the anti-cartel authorities. In fact, by doing so a firm would increase the probability of being prosecuted and found guilty — and receive a total payoff that is lower than its payoff from continuing to collude. The worst that can happen is for its rival to reveal information to the cartel office, but then its ultimate payoff does not depend on its action. Overall, mum is the word and the optimal strategy.

Now suppose that the leniency program is in effect. If my rival is going to come forward, then I am better off beating them to the cartel office. If in addition there is some probability that the cartel office will find out about my secret agreement from some other source, then even if my rival were not to go to the cartel office I would be better off by going to the cartel office to the extent that this will save me from punishment. All in all, the game may turn into a prisoner’s dilemma: although both firms would prefer to continue in the cartel agreement point, they both have as a dominant strategy to reveal information to the cartel office.

9.11. Corporate leniency. A study of the European corporate leniency program shows that the likelihood of a firm becoming the chief witness increases with its character as repeat offender, the size of the expected basic fine, the number of countries active in one group as well as the size of the firm’s share in the cartelized market. Are these results consistent with the discussion of these programs in the text?

Answer: Repeat offenders suffer bigger fines. This increases the incentive to be the leading witness, thus explaining the first result. The same argument applies to the size of the expected fine. The greater the number of countries involved, the greater the “prisoner’s dilemma” effect: if I don’t report, someone else will, so I might as well report.

9.12. Ivy League. The endowments of the Ivy League universities have increased significantly in recent decades. This wealth notwithstanding, for years the universities managed to refrain from using financial incentives as a means to compete for students: the manual of the council of Ivy League Presidents stated that the schools should “neutralize the effect of financial aid so that a student may choose among Ivy Group institutions for non-financial reasons.” In 1991, the Justice Department argued that this amounted to price collusion and forced the agreement to end. However, no significant price competition took place until 1998, when Princeton University started offering full scholarships for students with incomes below $40,000. Stanford, MIT, Dartmouth and Cornell followed suit. Allegedly, Harvard sent a letter to accepted 1998 applicants stating that “we expect that some of our students will have particularly attractive offers from the institutions with new aid programs, and those students should not assume that we will not respond.”

How do you interpret these events in light of the theories discussed in this chapter?

Answer: If the Department of Justice was right in assuming the council manual’s clause was an explicit form of price collusion, then what happened after 1991 is that collusion ceased to be explicitly supported by the clause and turned into tacit collusion. In fact, the

analysis in Chapter 8 suggests that explicit, contractual arrangements are not necessary to sustain a collusive agreement. The chapter also states that, under tacit collusion, each firm balances the short-run benefits from deviation against the long-term cost of entering into non-cooperative play. The fact that endowments have increased so much (especially Princeton’s) may be what has tipped the balance in the direction of giving away full scholarships.

\section*{9.13. Spanish hotels.} Based on data from the Spanish hotel industry, it was estimated that the rate set by hotel $i$ in market $k$ is positively influenced by a variable that measures the intensity of multimarket competition between hotel $i$ and its competitors in market $k$: the more markets $m \neq k$ in which firm $i$ and its competitors meet, the greater the measure of multimarket contact. It was also observed that the measure of multimarket contact is highly correlated with hotel chain size, that is, the larger hotel $i$’s chain, the greater the measure of multimarket contact for firm $i$.$^{33}$

Provide two interpretations for the positive coefficient of multimarket contact on hotel rates, one based on collusion, one based on a different effect.

\textbf{Answer:} When interaction between oligopolists takes place over a number of periods, it is easier to sustain collusion: long-term losses weigh more compared to short-term gains from deviation. Multimarket contact adds another “dimension” to the balance between gains and losses. A firm’s gain from deviating in one market may be punished by its competitors in all the markets they meet, making the potential cost from deviation higher. However, the optimal behavior of the deviating firm would call for deviation in all markets. Thus, we have higher losses from deviating but also higher gains. As discussed in Section 9.3, if everything is identical (firms, markets) then multimarket contact does not increase the likelihood of collusion because the potential gains from deviation increase in the same proportion as the losses. However, asymmetries between firms or markets can make losses weigh more than gains, thus increasing the likelihood of collusion. This justifies the positive correlation between multi-market contact and average rates.

There is, however, an alternative interpretation. Maybe rates are higher in hotels of greater size. This could happen either because consumers attach a greater value to hotels that have larger chains or because bigger hotel chains command greater (unilateral) market power. Given the empirical correlation between hotel size and multi-market contact this would also imply a correlation between multi-market contact and rates, even if there is no implicit or explicit collusion between hotel chains.

\section*{9.14. Railroads.} In 1986, the US Congress enacted a regulation (PL99-509) requiring railroads to disclose contractual terms with grain shippers. Following the passing of the legislation, rates increased on corridors with no direct competition from barge traffic, while rates decreased on corridors with substantial direct competition.$^{34}$ How do you interpret these events?

\textbf{Answer:} One possible interpretation for these results is that, when there is no competition to railroad shipping, there is potential for collusion among railroad operators, whereas the opposite is true when there is direct competition from barge traffic. In this context,
increased information about railroad contracts has the effect of

1. improving collusion among railroad operators when the latter have no competition. This is consistent with the idea that when price cuts are difficult to observe collusion is more difficult to sustain.

2. increasing competition in markets where railroad operators compete with barge operators. This is consistent with the idea that, in a competitive environment, better information about prices increases demand elasticity (consumer are more aware of price differences) and thus decreases margins.

9.15. Multi-market contact. Consider the model of multi-market contact presented in Section 9.3: Firm 1 has cost \(c\) in Market 1, while Firm 2 has a cost \(\bar{c}\). The situation is reversed in Market 2. Demand is the same both markets: consumers are willing to buy \(q\) units for a price of up to \(p^M\) (that is, for \(p \leq p^M\), quantity demanded does not depend on price). It is assumed that \(c < \bar{c} < p^M\). In each period, firms set prices in both markets simultaneously.

Determine the minimum value of the discount factor such that the optimal collusive solution is stable.

**Answer:** As discussed in Section 9.3, the efficient collusive agreement is the following: In each market, the firm with a cost advantage sets monopoly price, while the other sets a higher price and sells 0.

If firms price according to equilibrium strategies, then each firm gets 0 in the “foreign” market and \(q(p^M - c)\) in the “domestic market” in each period. The net present values of these profit streams is given by \(q(p^M - c)/(1 - \delta)\).

If a firm deviates from the equilibrium strategies and undercuts the rival in the rival’s “domestic” market, then it gets a total profit of \(q(p^M - c) + q(p^M - \bar{c})\) in the first period. Thereafter, it only makes a profit in the domestic market, specifically \(q(\bar{c} - c)\) per period.

It follows that the crucial equilibrium condition is given by

\[
\frac{q(p^M - c)}{1 - \delta} \geq q(p^M - c) + q(p^M - \bar{c}) + \delta \frac{q(\bar{c} - c)}{1 - \delta}
\]

Solving for \(\delta\), I get

\[
\delta \geq \frac{1}{2}
\]

**Challenging exercises**

9.16. Number of competitors. Consider an \(n\) firm homogeneous-good oligopoly with constant marginal cost, the same for all firms. Let \(\bar{\delta}\) be the minimum value of the discount factor such that it is possible to sustain monopoly prices in a collusive agreement. Show that \(\bar{\delta}\) is decreasing in \(n\). Interpret the result.

**Answer:** Let \(\pi^M\) be total industry profits. Under the collusive agreement, each firm receives \(\pi^M/n\). If one of the firms undercuts its rivals, then it gets approximately \(\pi^M\). Finally, if
firms revert to a (perpetual) price war each firm gets zero. It follows that the condition such that it is an equilibrium for firms to price at the monopoly level is given by

\[ \frac{1}{1 - \delta} \frac{\pi^M}{n} \geq \pi^M \]

Solving with respect to \( \delta \) we get

\[ \delta \geq \frac{n - 1}{n} \]

It follows that collusion is stable if and only if \( \delta > \bar{\delta} = \frac{n - 1}{n} \). (Note that the condition is independent of the value of \( \pi^M \), so the same condition would apply for any level of collusion.)

Taking the derivative of \( \bar{\delta} \) with respect to \( n \), we get

\[ \frac{d \bar{\delta}}{dn} = \frac{n - (n - 1)}{n^2} = 1/n^2 > 0 \]

It follows that \( \bar{\delta} \) is increasing in \( n \). In words, the more firms there are, the more difficult it is to sustain a collusive agreement. The idea is that the relative gain from cheating is greater the greater the number of firm (the profit from cheating is always the same, but the profit from collusion is lower the greater \( n \) is).

9.17. Two markets. Consider the model of multi-market contact presented in Section 9.3. Determine the minimum value of the discount factor such that the optimal collusive solution is stable.

**Answer:** The setting of the problem consists of Firms 1 and 2, and Markets A and B. Firm 1 has cost \( c \) in Market A, while Firm 2 has a cost of \( \bar{c} \). The situation is reversed in Market B. Demand is the same both markets. It is assumed that \( c < \bar{c} < p^M \).

As discussed in section 8.3 the efficient collusive agreement is the following: In each market, the firm with a cost advantage sets the monopoly price, while the other sets a higher price and sells 0. Let us use the following notation: \( \pi^M \) represents the monopoly profit of the firm with cost advantage, \( \pi^{M'} = \pi(p^M - \epsilon, \bar{c}) \) is the profit of the firm with high marginal cost when it charges (slightly less than) the monopoly price and \( \pi^C = \pi(\bar{c}, \ell) \) is the profit of the firm with low cost when it charges a price equal to the other firm’s costs.

In the efficient collusive agreement each firm gets:

\[ \pi^M + \delta \pi^M + \delta^2 \pi^M + ... = \pi^M \left( 1 + \delta + \delta^2 + ... \right) = \frac{1}{1 - \delta} \pi^M \]

If a firm decides to deviate, it will do so only in the market where it has a cost disadvantage, since in the other market it already earns monopoly profits. Suppose that the punishment for deviation is be for both firms to engage in a price war so that the prevailing price in each market is \( \bar{c} \). If Firm 2 deviates in Market A, then it gets \( \pi^{MC} \) in that market in the first period, plus 0 from then on; and \( \pi^M + \delta \pi^C + \delta^2 \pi^C + ... \) in the other market. The situation is symmetric. Therefore, the deviating firm’s total profits are given by:

\[ \pi^{MC} + \pi^M + \delta \pi^C + \delta^2 \pi^C + ... = \pi^{MC} + \pi^M + \frac{\delta}{1 - \delta} \pi^C \]
The stability condition is the following:

\[
\frac{1}{1 - \delta} \pi^M \geq \pi^{MC} + \pi^M + \frac{\delta}{1 - \delta} \pi^C
\]

This gives the minimum value for the discount factor:

\[
\delta = \frac{\pi^{MC}}{\pi^{MC} + \pi^M - \pi^C}
\]

\section*{9.18. Secret price cuts.} This exercise formalizes the model of secret price cuts presented in Section 9.2.\textsuperscript{35} Suppose that all consumers are willing to pay \( u \) for the (homogeneous) product sold by two duopolists. In each period, demand can be high (probability \( 1 - \alpha \)) or low (probability \( \alpha \)). When demand is high, \( h = 1 \) units can be sold at price \( u \) (or any lower price). When demand is low, only \( l = 0 < h \) units can be sold. The probability that demand is high or low in each period is independent of what it was in the previous period. Moreover, firms are unable to observe the state of market demand; all they can observe is whether their own demand is high or low. Finally, for simplicity, assume that production costs are zero.

Consider the following equilibrium strategies. Firms start by setting \( p = u \). If they receive a positive demand (namely, \( \frac{1}{2} \)), then they continue to set \( p = u \), that is, they remain in the “co-operative phase”. If however one of the firms (or both) receives zero demand, then both firms enter into a “price war”: they set \( p = 0 \) during \( T \) periods and, after this period, revert to \( p = u \) again (the co-operative phase).\textsuperscript{9} Let \( V \) be the net present value of a firm in equilibrium (starting in a period where collusion takes place).

(a) Show that

\[
V = (1 - \alpha) \left( \frac{u}{2} + \delta V \right) + \alpha \delta^{T+1} V
\]

Answer: The first term on the right-hand side corresponds to the case when demand is high (probability \( 1 - \alpha \)), whereas the second term corresponds to the case of low demand (probability \( \alpha \)). If demand is high, then each firm receives current profits of \( \frac{u}{2} \). Moreover, beginning next period, their continuation expected payoff is \( V \), for there is no reason to start a price war. If, however, demand is low (probability \( \alpha \)), then it is common knowledge that at least one of the firms receives zero demand, and that a price war will start in the next period. As a result, firms receive zero profits today (because demand is zero) and zero profits in the next \( T \) periods (because they engage in a price war). After these \( T \) periods, firms revert to the co-operative phase, so that their continuation expected payoff from then on is \( V \).

(b) Show that the best a firm can do by deviating is to get

\[
V' = (1 - \alpha) u + \delta^{T+1} V
\]

\textsuperscript{9} Notice that, if a firm receives zero demand, then it is common knowledge that a price war is going to start, that is, it is common knowledge that one of the firms receives zero demand. In fact, either demand is low, in which case both firms receive zero demand, or one of the firms deviates from \( p = u \), in which case the deviating firm knows that the rival receives zero demand.
Answer: In words, this equation tells us that, if demand turns out to be high (probability $1 - \alpha$), then setting a slightly lower price gives the deviator a current profit of $u$ (as opposed to $\frac{u}{2}$). However, regardless of what the state of demand is today, firms will certainly enter in a price war beginning in the next period. In fact, regardless of the state of demand, the rival firm (the non-deviator) receives zero demand today, the condition that triggers a price war. For this reason, expected future discounted payoff is simply $\delta^{T+1} V$.

(c) Show that the condition that the prescribed strategy constitutes an equilibrium is given by

$$1 \leq 2 \left(1 - \alpha\right) \delta + \left(2 \alpha - 1\right) \delta^{T+1}$$

Answer: The condition is given by $V \geq V'$. It simplifies into the condition in the text.

(d) What is then the optimal equilibrium?

Answer: If $\alpha < \frac{1}{2}$, then the above condition is equivalent to $T \geq \overline{T}$, where $\overline{T}$ is a positive number. What is then the optimal equilibrium? (Note that optimality here is understood within the class of equilibria we are considering. It is possible to find collusive equilibria that perform better than the ones considered here.) In other words, what is the optimal value of $T$? From the equation for $V$, we can see that the equilibrium discounted payoff is decreasing in the value of $T$. This is intuitive, for the greater the value of $T$ the longer the price wars will be; and firms don’t like price wars. Therefore, the optimal value of $T$ is the lowest value such that the equilibrium is stable, that is, the lowest value such that the condition $V \geq V'$ holds, that is, $T = \overline{T}$.

**9.19. Demand fluctuations.** This exercise formalizes the model of demand fluctuations considered in Section 9.2. The new model is similar to the model in Exercise 9.18, with the difference that we now assume that in each period, before setting prices, firms observe the state of demand. We also make the simplifying assumption that $\alpha = \frac{1}{2}$, that is, the high- and the low-demand states are equally likely.

(a) Show that, if the discount factor is sufficiently large, specifically, if $\delta > \frac{2h}{3h+l}$, then there exists an equilibrium where firms set monopoly price in every period (similarly to Section 9.1).

Answer: In this equilibrium, firms set $p = u$ in every period, regardless of the state of demand. In fact, if a firm decides to go along with the agreed-upon equilibrium strategies, its expected discounted profits are

$$\frac{1}{2} ud + \frac{\delta}{1 - \delta} \left(\frac{1}{2} uh + \frac{1}{2} ul\right)$$

where $d = h$ or $d = l$ depending on whether today’s demand is high or low, respectively. Assuming that a deviation implies that firms switch to setting price equal to marginal cost, the payoff from deviation is simply $ud$, where, again, $d = h$ or $d = l$. The conditions that

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1. We are ignoring here the fact that $T$ must be an integer.
2. Firms can also observe past decisions by rival firms.
setting \( p = u \) is an equilibrium are therefore
\[
\frac{1}{2} u h + \frac{\delta}{1 - \delta} \left( \frac{1}{2} u h + \frac{1}{2} ul \right) > uh
\]
\[
\frac{1}{2} ul + \frac{\delta}{1 - \delta} \left( \frac{1}{2} u h + \frac{1}{2} ul \right) > ul
\]
or simply
\[
\delta > \frac{2}{3 + l/h} \equiv \bar{\delta}
\]
\[
\delta > \frac{2}{3 + h/l} \equiv \delta
\]
(9.6)

Notice that, since \( h > l \), (9.6) implies (??). This is intuitive: the temptation to cheat on the agreement and set a slightly lower price is especially strong in periods of high demand. The condition for stability of the full-collusion agreement is therefore (9.6).

(b) Suppose now that the discount factor \( \delta \) is lower than, but close to, the threshold derived in the previous answer. Show that, while there exists no equilibrium where firms set monopoly price in every period, there exists an equilibrium where firms set monopoly price during periods of low demand and a lower price during period of high demand.

**Answer:** If \( \delta \) is lower than, but close to, \( \bar{\delta} \), then full collusion cannot be an equilibrium. However, suppose that firms decide to set price equal to \( p_h < u \) during periods of high demand and \( p_l = u \) during periods of low demand. The condition for no-deviation during periods of high demand is now given by
\[
\frac{1}{2} p_h h + \frac{\delta}{1 - \delta} \left( \frac{1}{2} p_h h + \frac{1}{2} ul \right) \geq p_h h
\]
or simply
\[
p_h < \frac{\delta l/h}{2 - 3 \delta} u
\]
Since firms would like to set prices as high as possible, we get
\[
p_h = \frac{\delta l/h}{2 - 3 \delta} u
\]
(9.7)

Substituting \( \bar{\delta} \) for \( \delta \) in (9.7), we get, as expected, \( p = u \). Any value \( \delta < \bar{\delta} \) yields \( p_h < u \). Assuming that \( \delta \) is close to \( \bar{\delta} \), \( p_h \) will in turn be close to \( u \). This implies that the constraint (??) is still satisfied. We thus conclude that setting \( p = p_h \) in periods of high demand and \( p = u \) in periods of low demand is indeed an equilibrium.

**Applied exercises**

9.20. Multimarket contact. Choose a pair of firms from a given industry. Determine the extent to which there is multi-market contact between these firms and how it may help them soften competition.