Answers to Chapter 3 Exercises

Review and practice exercises

3.1. DRAM factory. You own and operate a facility located in Taiwan that manufactures 64-megabit dynamic random-access memory chips (DRAMs) for personal computers (PCs). One year ago you acquired the land for this facility for $2 million, and used $3 million of your own money to finance the plant and equipment needed for DRAM manufacturing. Your facility has a maximum capacity of 10 million chips per year. Your cost of funds is 10% per year for either borrowing and investing. You could sell the land, plant and equipment today for $8 million; you estimate that the land, plant, and equipment will gain 6% in value over the coming year. (Use a one-year planning horizon for this problem.)

In addition to the cost of land, plant, and equipment, you incur various operating expenses associated with DRAM production, such as energy, labor, raw materials, and packaging. Experience shows that these costs are $4 per chip, regardless of the number of chips produced during the year. In addition, producing DRAMs will cause you to incur fixed costs of $500k per year for items such as security, legal, and utilities.

(a) What is your cost function, \( C(q) \), where \( q \) is the number of chips produced during the year?

Answer: The $5 million you originally spent for the land, plant, and equipment is a sunk expenditure and thus not an economic cost. However, there is a “user cost of capital” associated with the land, plant and equipment, based on its current market value of $8 million and your cost of funds and the rate of depreciation or appreciation of the asset over the planning horizon. Your (opportunity) cost of investing $8 million for one year is $800k, but these assets will appreciate by $480k over the year, giving a (net) user cost of capital of $320k. (The depreciation rate is 6%.) This is a fixed cost of making DRAM’s, to which we must add the other fixed costs of $500k to get a combined fixed cost of $820k for the year. The variable costs are a constant $4 per chip, so the cost function is \( C(Q) = 820k + 4Q \), in the range of \( 0 < Q < 1m \). (One could also report that \( C(0) = 0 \), by definition, and that \( C(Q) \) is infinite for \( Q > 1m \), since your maximum capacity is one million chips per year. Of course, in practice there would likely be a way to push production beyond “rated capacity,” at some cost penalty, but that is beyond the scope of this problem.)
Assume now that you can sell as many chips as you make at the going market price per chip of $p$.

(b) What is the minimum price, $p$, at which you would find it profitable to produce DRAMs during the coming year?

**Answer:** The average cost function is $AC(Q) = 820k/Q + 4$, again up to one million chips per year. This declines with $Q$, so the minimum $AC$ is achieved at full capacity utilization. At one million chips per year, the fixed costs come to $0.82$ per chip, so average costs are $4.82$ per chip. This is your minimum average cost, and thus the minimum price at which it makes sense to stay open for the year.

**3.2. mp34u.** Music Ventures sells a very popular mp3 player, the mp34u. The firm currently sells one million units for a price of $100$ each. Marginal cost is estimated to be constant at $40$, whereas average cost (at the output level of one million units) is $90$. The firm estimates that its demand elasticity (at the current price level) is approximately $-2$. Should the firm raise price, lower price, or leave price unchanged? Explain.

**Answer:** Optimal pricing implies $m = \frac{1}{|\epsilon|}$. In this problem, we have

$$m = \frac{p - MC}{p} = \frac{100 - 40}{100} = 0.6,$$

which is greater than $1/|\epsilon - 2| = 0.5$. This tells us that the price/cost margin is too high, so a lower price would be optimal. Note that the margin depends on $MC$, not $AC$.

**3.3. KindOfBlue jeans.** Two years ago, KindOfBlue jeans were priced at $72$ and $121,000$ units were sold. Last year, the price was lowered to $68$ and sales increased to $132,000$.

(a) Estimate the value of the demand elasticity.

**Answer:** Using the percent variation method (and measuring $q$ in 000), we have

$$\epsilon = \frac{132 - 121}{121} \approx -1.64$$

Using the logarithm method, we have

$$\epsilon = \frac{\ln 132 - \ln 121}{\ln 68 - \ln 72} \approx -1.52$$

(b) Based on your estimate of the demand elasticity, how many units would you expect to be sold if price were lowered by an additional $\$1$?

**Answer:** Let $q$ be the new value of $q$. Continuing with the first method, I would estimate

$$q - 132 \approx -1.636 \times \frac{67 - 68}{68} \times 132 \approx 135,177$$
Using the logarithm method,

\[
\ln(q) - \ln(132) = -1.522 \times (\ln(67) - \ln(68)) \\
q = \exp\left(\ln(132) - 1.522 \times (\ln(67) - \ln(68))\right) \approx 135,011
\]

(c) In order to increase profits, should price be lowered below $68? If your answer begins — as it should! — with “it depends,” indicate as clearly as possible what additional information you would need and how you would base your answer on such additional information.

**Answer:** It depends on what the value of marginal cost is. To be more precise, if marginal cost is lower than marginal revenue, then we should further increase output, that is, decrease price. Marginal revenue is given by

\[
MR = p \left(1 + \frac{1}{\epsilon}\right)
\]

Depending on which value of elasticity we use, we get \(MR \approx 26.5\) or \(MR \approx 23.3\) (when \(p = 68\)). So, if marginal cost is lower than 23.3 we should further decrease price. If marginal cost is greater than 26.5, we should instead increase price. Otherwise, it is not clear.

**3.4. EZjoint.** After spending 10 years and $1.5 billion, you have finally gotten Food and Drug Administration (FDA) approval to sell your new patented wonder drug, which reduces the aches and pains associated with aging joints. You will market this drug under the brand name of EZjoint. Market research indicates that the demand elasticity for EZjoint is -1.25 (at all points on the demand curve). You estimate the marginal cost of manufacturing and selling one more dose of EZjoint is $1.

(a) What is the profit-maximizing price per dose of EZjoint?

**Answer:** The optimal pricing rule (one version of it) states that

\[
p = \frac{MC}{1 + \frac{1}{\epsilon}}
\]

where \(\epsilon\) is the price elasticity of demand. We thus have

\[
p = \frac{1}{1 - \frac{1}{1.25}} = 5
\]

Note that R&D expenditures are a sunk cost and thus do not enter into the pricing decision.

(b) Would you expect the elasticity of demand you face for EZjoint to rise or fall when your patent expires?

**Answer:** The level of demand for EZJoint must fall now that there are many very close substitutes in the form of generic versions. Hopefully, your brand will still allow you to command a premium price, but surely at any given price you will sell less as a result of the presence of the generic competition.
The elasticity of demand for EZjoint will very likely rise now that closer substitutes are available. Customers will presumably be more price sensitive, which in turn induces you to set a lower price. However, to the extent that the price elasticity is lower at higher price levels of the demand curve; and to the extent that competitions from generics forces the branded drug firm to specialize on the high-valuation segment; then it is possible that the elasticity faced by the former monopolist now be less elastic.

Suppose that, after patent expiry, a generic version of EZjoint was introduced in the market (under the chemical name clorophosphatane). Reacting to entry, EZjoint decided to increase price.

(c) Can this behavior be consistent with rational profit maximizing?

**Answer:** One possible explanation is that the market is divided into two segments, one more price sensitive than the other. While a monopolist, EZjoint was setting a price that balanced the goal of achieving both market segments. Once entry by a generic manufacturer has taken place, EZjoint decided to “give up” on the price-sensitive segment and target exclusively the segment with more inelastic demand — thus the price increase.

## 3.5. Las-O-Vision

Las-O-Vision is the sole producer of holographic TVs, 3DTVs. The weekly demand for 3DTVs is \( D(p) = 10200 - 100p \). The cost of producing \( q \) 3DTVs per week is \( q^2/2 \) (note this implies that \( MC = q \)).

(a) What is Las-O-Vision’s total revenue schedule?

**Answer:** Total Revenue is given by \( p(q)q \), that is, the revenue that Las-O-Vision receives when it sells \( q \) units. To get \( p(q) \), we invert the demand function \( q = 10.2k - 100p \) by solving for \( p \) in terms of \( q \), or \( p(q) = 102 - q/100 \). Substituting this into our total revenue equation, we obtain \( TR(q) = (102 - q/100)q = 102q - q^2/100 \).

(b) What is Las-O-Vision’s marginal revenue schedule?

**Answer:** Marginal revenue is the derivative of Total Revenue with respect to \( q \), so \( MR(q) = 102 - q/50 \); or, since our demand equation is linear in \( q \), we can obtain it by recalling that the marginal revenue curve is twice as steep as the inverse demand curve and starts at the same point on the vertical axis.

(c) What is the profit-maximizing number of 3DTVs for Las-O-Vision to produce each day? What price does Las-O-Vision charge per 3DTV? What is its daily profit?

**Answer:** The profit maximizing quantity, \( q^* \), is that quantity at which marginal cost and marginal revenue are equal. Setting \( MR(q) = MC \), we have \( 102 - q^*/50 = q^* \), or \( q^* = 100 \). The profit maximizing price is that which generates \( q^* = 100 \) in sales or, substituting into the inverse demand function calculated in (a), \( p(100) = 102 - (100/100) = 101 \). When selling 100 units, Las-O-Vision generates Total revenues equal to \( TR(100) = 102 \times 100 - 1002/100 = 101.1k \). Its total cost is \( 100^2/2 = 5k \). Therefore its total profit when it sells 100 units is \( 10.1k - 5k = 5.1k \).
Table 3.2
Monsanto’s Roundup: price, sales, revenue and profit

<table>
<thead>
<tr>
<th>Year</th>
<th>Price</th>
<th>Quantity</th>
<th>Revenue</th>
<th>Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>1995</td>
<td>45</td>
<td>13</td>
<td>599</td>
<td>400</td>
</tr>
<tr>
<td>1996</td>
<td>44</td>
<td>16</td>
<td>718</td>
<td>473</td>
</tr>
<tr>
<td>1997</td>
<td>40</td>
<td>20</td>
<td>823</td>
<td>517</td>
</tr>
<tr>
<td>1998</td>
<td>35</td>
<td>28</td>
<td>1003</td>
<td>578</td>
</tr>
<tr>
<td>1999</td>
<td>33</td>
<td>33</td>
<td>1092</td>
<td>592</td>
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<tr>
<td>2000</td>
<td>28</td>
<td>40</td>
<td>1105</td>
<td>509</td>
</tr>
<tr>
<td>2001</td>
<td>25</td>
<td>45</td>
<td>1139</td>
<td>464</td>
</tr>
</tbody>
</table>

3.6. Monsanto. With reference to the discussion in Box 3.1:

(a) How do you know that cutting the price of Roundup was a good idea for Monsanto?

**Answer:** The key issue here is elasticity: Does a lower price produce greater revenue and profit? The case suggests it does. In order to estimate whether profit increased when price fell, you can guesstimate revenue (take price and quantity from the figures) and cost (the international price is an upper bound). Almost any estimate will show profit going up. Table 3.2 is based on a marginal cost of 15, which is probably on the high side (and thus understates the increase in profit). A more specific, quantitative answer is given in the next answer.

(b) How might you estimate the elasticity of demand and the profit-maximizing price for 1995. Do you think Monsanto set the right price?

**Answer:** The first issue to address is that the price drop is not a controlled experiment: other things might have been changing at the same time. Did the market for herbicides overall grow? We don’t know. Once you’ve made this point, you can think about estimating the value of elasticity by applying the ratio of variations method for year-on-year variations. Some estimates can be found in Table 3.3.

If you take 2.5 as a ballpark estimate, the ratio of price to marginal cost should be 1.67, so with a cost of 15 the price should be about 25, that is, lower than what Monsanto chose. There is an important qualification to this estimate of optimal price: One possibility is that there is some form of price discrimination going on here. At the time this case takes place, Monsanto was also selling Roundup Ready seeds, which is a complementary product to Roundup. If seed buyers were willing to pay more than other users of Roundup, Monsanto could target both markets by charging less for Roundup and more for the seeds. Some observers suggest this is precisely what Monsanto did. More on this in Chapter 6.

(c) If cutting price was a good idea, why didn’t Monsanto do it earlier?

**Answer:** While we have data looking at the real effect of the price cut, Monsanto was
Table 3.3
Monsanto’s Roundup: demand elasticity

<table>
<thead>
<tr>
<th>Year</th>
<th>Elasticity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1996</td>
<td>-7.13</td>
</tr>
<tr>
<td>1997</td>
<td>-2.59</td>
</tr>
<tr>
<td>1998</td>
<td>-2.51</td>
</tr>
<tr>
<td>1999</td>
<td>-2.11</td>
</tr>
<tr>
<td>2000</td>
<td>-1.07</td>
</tr>
<tr>
<td>2001</td>
<td>-1.32</td>
</tr>
</tbody>
</table>

working off estimates, which may have suggested that what they were doing was optimal given the information available. Even if Monsanto had an estimate of demand elasticity similar to the ones we computed, there are several considerations to take into account. First, it may have been the case that Monsanto’s executives had never thought about changing prices (this possibility is not very appealing to an economist, who always assumes that firms maximize profits). Second, we know that success breeds laziness: while their product was successful and profits high Monsanto’s executives had little incentive to work harder. Finally, one plausible explanation for waiting until the mid-1990s is risk: experimenting with different prices might be costly. Early on during the Roundup patent life, there is a lot at risk. When the patent is about to expire, the costs of making a mistake are lower. In other words, “If it ain’t broke don’t fix it; and if it’s about to break, try fixing it now.”

### 3.7. Windows

Microsoft is the dominant player in the market for desktop operating systems. Suppose each copy of Windows is sold for $50. Suppose moreover that the marginal cost of production and shipping is $5. What value of the demand elasticity is consistent with this price? Does it make sense? What elements of the OS market may be missing from the elasticity rule?

**Answer:** In order for the price of $50 to be consistent with the simple formula for optimal pricing, the price elasticity of demand, $\varepsilon$, would need to be such that

$$
\frac{50 - 5}{50} = \frac{1}{-\varepsilon}
$$

which implies $\varepsilon \approx -1.11$. This means that, if the price of Windows were to go up by 10%, more than 10% would stop buying Windows. But 10% of $50 is $5, which is a very small fraction of the overall price tag a consumer pays for a computer. It seems unreasonable that so many users would move away from Windows (to a Mac? Linux?) because of such a small price increase.

To put it differently: if Microsoft were to apply the optimal price formula with the true value of elasticity, it would probably set a higher price for Windows. Why then such a “small” price for the Windows operating system? One possible answer is that, by selling more units of Windows, Microsoft is able to sell more of other products (Office?), so that there is some cross-subsidization going on. Another possibility is that, although the short
run elasticity is small, in the long run setting a higher price risks “tipping” the industry towards a different operating system. More on this in Chapter 16.

3.8. Profit maximization. Explain why the assumption of profit maximization is or is not reasonable?

Answer: The answer to this question may be found in Section 3.3. The main reason why we might think that the assumption of profit maximization is not reasonable is that firm managers are frequently not the firm owners; and the goals of managers frequently differ from those of the owners. However, it can be argued that the discipline imposed by shareholders, the labor market, the product market and the capital market are sufficient to enforce profit maximization. In particular, the threat of a takeover has been found to have a significant effect on value maximization.

3.9. Car parts. Two parts in an automobile taillight are the plastic exterior cover and the light bulb. Which of these parts is a car company more likely to manufacture in-house? Why?

Answer: Light bulbs are generally a homogeneous good. External suppliers enjoy economies of scale and specialization; and supply the entire industry. In contrast, the plastic exterior cover must be custom-designed and manufactured for each make and model. Because it requires more Relationship Specific Investment (RSI), it is more likely to be made in-house.

3.10. Jet engines. There are three main suppliers of commercial jet engines, Pratt & Whitney, General Electric, and Rolls-Royce. All three maintain extensive support staff at major (and many minor) airports throughout the world. Why doesn’t one firm service each airport? Why do all three feel they need to provide service and support operations worldwide themselves? Why don’t they subcontract this work? Why don’t they leave it entirely to the airlines?

Answer: Jet engines are marvelously idiosyncratic. The knowledge, tools and parts needed to service one family (brand) of engines do not transfer fully across brands. One firm does not typically service each airport because the economies of scale (across brands) are small and the economies of specialization (within brand) are large. The only thing worse for an airline than an AOG (an aircraft sitting on the ground with a broken engine) is an aircraft flying with a broken engine or two. To ensure their reputation and revenues and to avoid ex post hold up, airlines demand before purchasing an aircraft that engine makers pre-commit capital to ensure that parts and service are available at major stations world-wide. Because the skills to do this are RSIs, and because the engine owner’s reputation is at stake, to sell engines and credibly commit to keeping them running, each manufacturer must provide service and support at major stations.

Subcontracting would be difficult because of the RSI required (the subcontractor would fear hold-up) and because a poor subcontractor would impose a negative externality on the manufacturer. When the jet goes down, the manufacturer’s reputation will suffer on a scale beyond any contractual penalty a subcontractor could likely be held to, so the work is not usually subcontracted. In addition, the manufacturers benefit directly from direct feedback
within the firm on the performance of the engines they produce. This information may flow more readily within the firm than across firms.

Some airlines with sufficient scale do perform their own routine engine maintenance at their own maintenance bases. However, the airlines cannot efficiently do emergency engine repairs away from an airline’s main bases. While there are enough GE engines going through Karachi International Airport to justify an on-site GE technical support staff, most airlines do not have enough flights through Karachi to justify the investment. The economies of scale in non-routine work are site and engine specific, not generally airline specific.

3.11. Smart car. The Smart car was created as a joint venture between Daimler-Benz AG and Swatch Group AG. Although Micro Compact Car AG (the name of the joint venture) was originally jointly owned, in November of 1998 Daimler-Benz AG took complete control by buying Swatch’s share. The deal put an end to a very stressed relationship between Daimler and Swatch. What does Section 3.4 suggest as to what the sources of strain might have been?

Answer: Section 3.4 suggests that, when two parties invest in specific assets and contracts are incomplete, the equilibrium solution is inefficient in every situation short of vertical integration. It is likely that some of this happened in the “stressed relationship” between Daimler and Swatch. Since none of the parties was in complete control (and ownership) of the future developments in the joint venture, the incentives for each party to invest were less than efficient.

3.12. Franchise retailing. Empirical evidence from franchise retailing suggests that, even when stores have similar characteristics, the mother company resorts to a mix between company-owned stores and franchised ones. How can this be justified?

Answer: Franchisers face a problem in judging the performance of their franchisees. Keeping some retail locations in-house provides the parent company with a baseline of more readily accessible and less biased information against which the performance of the franchisees can be measured. This information then helps to set standards in negotiating and administering future franchise contracts. Franchising the majority of retail locations limits the parent’s direct financial outlay and exposure. Franchisers might also have an interest in direct control of locations that could have a particularly strong impact on its brand or reputation.

3.13. Body Shop. The U.K. Body Shop franchise network consists of three types of stores: franchised, company owned and partnership stores. All stores that are distant from headquarters by more than 300 miles are franchised. More than half of the company-owned stores are within 100 miles of headquarters. How can you explain these facts?

Answer: Owning a store has the advantages of vertical integration discussed in Section 3.2. However, it also has the problem that it requires increased monitoring by the store owner. We would expect the costs from monitoring to be lower the closer the store is to headquarters. Consequently, we would expect vertical integration to be more likely when the store is located closer to headquarters. The empirical evidence seems consistent with
this hypothesis.

3.14. Intel. Explain why Intel has maintained, if not increased, its competitive advantage with respect to rivals. Indicate the explanatory power of the different causes considered in the text (impediments to imitation, causal ambiguity, strategy, history).

Answer: This is a complex question. In fact, as argued in this chapter, this is the question in strategy. A good source for the particular case of Intel is the HBS case “Intel Corporation: 1968–1997,” No. 9–797–137 (Rev. October 21, 1998).

Challenging exercises

3.15. Input shares. Consider a firm with production function (Cobb-Douglas)

\[ q = \xi K^\alpha L^\beta \]

Suppose that output price \( p \) and input prices \( r, w \) are given. Show that a profit maximizing firm chooses inputs levels such that the ratio between labor costs and total revenue \( w L/pq \) is equal to \( \beta \).

Answer: Firm profit is equal to

\[ p\xi K^\alpha L^\beta - (rK + wL) \]

The first-order condition for maximization with respect to \( L \) is given by

\[ \beta p\xi K^\alpha L^{\beta-1} - w = 0 \]

Noting that

\[ \xi K^\alpha L^{\beta-1} = q/L \]

the first-order condition may be re-written as

\[ \beta pq/L - w = 0 \]

or simply

\[ \beta = \frac{wL}{pq} \]

3.16. Cost minimizing input mix. Consider a firm with production function (Cobb-Douglas)

\[ q = K^{\frac{1}{2}} L^{\frac{1}{2}} \]

Suppose that one unit of capital costs \( r = 12.5 \), whereas one unit of labor costs \( w = 8 \).

(a) Determine the optimal input mix that leads to an output of \( q = 2 \).

Answer: The firm’s problem is

\[ \min_{K,L} rK + wL \]
subject to
\[ q = K^{\frac{1}{2}} L^{\frac{1}{2}} \]

Solving the constraint for \( K \) I get
\[ K = \frac{q^2}{L} \quad (3.4) \]

Substituting this for \( K \) in the objective function, the problem becomes
\[ \min_L r \frac{q^2}{L} + w L \quad (3.5) \]

The first-order condition for this minimization problem is given by
\[ -r \frac{q^2}{L^2} + w = 0 \]

Solving for \( L \), I get
\[ L = q \sqrt{\frac{r}{w}} \quad (3.6) \]

Substituting 2, 12.5 and 8 for \( q, r \) and \( w \) I get \( L = 2.5 \). Substituting 2.5 for \( K \) in (3.4), I finally get \( K = 1.6 \)

(b) Determine the firm’s cost function, that is, the minimum cost required to produce output \( q \).

Answer: From (3.6) for \( L \) in (3.5) and simplifying I get
\[ C = (2 \sqrt{r w}) q \]

Substituting 12.5 and 8 for \( r \) and \( w \), respectively, we get \( C = 20 q \).

3.17. Pricing with capacity constraints. Consider again the case of a monopoly facing linear demand and constant marginal cost. Demand is
\[ q = a - bp, \]

and marginal cost is \( MC = c \). Suppose moreover that the monopolist has a limited capacity of \( K \). In other words, it must be \( q \leq K \). What is the optimal price?

Answer: The easiest way to solve this problem is to first derive the optimal output level under the assumption that there are no capacity constraints; and then compare the candidate output level to the capacity constraint. Solving the demand curve for \( p \), we have
\[ p = \frac{a}{b} - \frac{1}{b} q \]

This leads to a profit function
\[ \pi(q) = p q - c q = \left( \frac{a}{b} - \frac{1}{b} q \right) q - c q \]
The first-order condition for profit maximization (ignoring capacity constraints) is given by

\[-\frac{1}{b} q + \frac{a}{b} - \frac{1}{b} - c = 0,
\]
or simply

\[q = \frac{a - b c}{2}\]

Now we have two possibilities. If \(\frac{a-b c}{2} < K\), then \(q = \frac{a-b c}{2}\) is the optimal output level. The optimal price is then given by

\[p = \frac{a}{b} - \frac{1}{b} \left( \frac{a-b c}{2} \right) = \frac{a + b c}{2 b}\]

If however \(\frac{a-b c}{2} > K\), then the optimal output level is \(q = K\). The reason is that marginal revenue is greater than marginal cost up to \(q = \frac{a-b c}{2}\), and so, if \(q < \frac{a-b c}{2}\) then we want \(q\) as high as possible. Finally, in this case optimal price is given by

\[p = \frac{a}{b} - \frac{1}{b} K = \frac{a - K}{b}\]

### 3.18. Optimal bidding

You are one of two companies bidding to try to win a large construction project. Call your bid \(b\). You estimate that your costs of actually performing the work required will be $800k. You are risk neutral.\(^j\) You will win if and only if your bid is lower than that of the other bidder. You are not sure what bid your rival will submit, but you estimate that the rival’s bid is uniformly distributed between $1m and $2m.\(^k\) What bid should you submit?

**Answer:** A risk-neutral bidder will use a bidding strategy that maximizes the expected value of its bid \(b\). This entails picking a bid value \(b\) that balances two offsetting effects: changes in the value of winning due to changes in the bid (the larger your bid is the more valuable the contract is) and changes in the chances of winning due to changes in \(b\) (the larger your bid is the less likely you will win). Formally, the expected value of from placing a bid \(b\) can be expressed

\[E[b] = (b - 800k) \times \mathcal{P}(b < b_r)\]

where \(b_r\) is the rival bidder’s bid and \(\mathcal{P}(b < b_r)\) is the probability that its bid, \(b\), is less than its rival’s bid, \(b_r\). The first term in this equation is simply the payoff when a bidder wins. The second term is its chances of winning (which requires that \(b < b_r\)).

In this problem, the focal bidder believes that its rival’s bid can be anywhere between $1m and $2m so

\[\mathcal{P}(b < b_r) = 1 - (b - 1m)/1m\]

\(^j\) We say that an agent is risk neutral if he or she is indifferent between receiving 100 for sure and receiving 0 or 200 with probability 50% each. More generally, a risk-neutral agent only cares about the expected value of each outcome.

\(^k\) By “uniformly distributed between \(a\) and \(b\)” we mean that all values between \(a\) and \(b\) are equally likely.
for all bids between $1m and $2m. (Note that \( P(b < b_r) = 1 \) for all bids less than $1m since the focal bidder believes that the rival never bids below $1m; and \( P(b < b_r) = 0 \) for all bids greater than $2m since the focal bidder believes that the rival never bids above $2m.) Substituting this expression into the expected value from placing a bid \( b \) we obtain

\[
E[b] = (b - 800k)
\left(1 - \frac{b - 1m}{1m}\right)
\]

From this expression it is clear that the bidder’s payoff, conditional on winning the bid, goes up with \( b \); but that its chance of winning declines with \( b \). Picking the optimal \( b \) entails finding the maximum of \( E[b] \), which we can easily obtain by taking the derivative of \( E[b] \), setting it equal to zero and solving for \( b \). This bid will be the point at which the two effects of changing \( b \) just offset each other. Working in millions as units, we have

\[
\frac{\partial}{\partial b} E[b] = (1 - (b - 1)) - (b - .8) = 0
\]
\[
b - .8 = 1 - b + 1
\]
\[
b = 1.4m
\]

An alternative approach to this problem is to construct a demand function from the information we have about the market. You can then solve the problem in the same way as you would with more straightforward problems in which you are given an explicit demand function (i.e., set \( MR = MC \) and solve for optimal \( q \), then solve for optimal \( b \)). To see how this approach works, note that the bid the firm submits is just like a price. The higher its bid, the lower its expected demand will be. In this case, demand falls as the price goes up because the firm’s chance of winning is falling. Formally, expected demand, \( q \), at any level \( b \) is equal to

\[
q = 1 - P(b < b_r) = 1 - (b - 1m)/1m = 2 - b/1m
\]

As this equation indicates, when the firm’s bid is equal to 1m, “demand” comes to 1 unit. That is, the firm is sure to win the contract. As the bid (the price) increases, demand falls to some fraction of a unit until at 2m “demand” is zero. Since the contract is a winner-take-all item, the idea of fractional units is not really correct, but if there were, say, \( N \) consumers instead of a single consumer, and the firm was bidding against other firms for the business of each consumer, the aggregate demand function would then be

\[
q_N = N \left(2 - \frac{b}{1m}\right) = 2N - \frac{bN}{1m}
\]

This is like a simple linear demand function.

To continue with this approach, we need to invert the demand function and solve for \( b \) to get

\[
b = 2m - 1m \times q
\]

The bidder’s total revenue is then

\[
bq = (2m - 1m \times q) q
\]
Taking the derivative of this total revenue function, we find that the marginal revenue of
the firm is
\[ 2m - 2 \times 1m \times q \]
As we would expect, the marginal revenue curve has twice the slope of the inverse demand
curve. We can then set this marginal revenue equal to the marginal cost of 800,000 to get
optimal \( q \):
\[
800k = 2m - 2 \times 1m \times q \\
2 \times 1m \times q = 1.2m \\
q = 1.2/2 = .6
\]
Substituting this value into our inverse demand function, we obtain the optimal bid of
\[ b = 2m - .6 \times 1m = 1.4m \]

3.19. Competitive pressure. Suppose that a firm’s profits are given by \( \pi = \alpha + \phi(e) + \epsilon \),
where \( \alpha \) denotes the intensity of product market competition, \( e \) effort by the manager, and
\( \epsilon \) a random shock. The function \( \phi(e) \) is increasing and concave, that is, \( d\phi/de > 0 \) and
\( d^2\phi/de^2 < 0 \).

In order for the firm to survive, profits must be greater than \( \bar{\pi} \). The manager’s payoff
is \( \beta > 0 \) if the firm survives and zero if it is liquidated, that is, if profits fall short of the
minimum target. The idea is that if the firm is liquidated, then the manager loses his or
her job and the rents associated with it.

Suppose that \( \epsilon \) is normally distributed with mean \( \mu \) and variance \( \sigma^2 \), and that \( \mu > \bar{\pi} \).
Show that increased product market competition (lower \( \alpha \)) induces greater effort by the
manager, that is, \( de/d\alpha < 0 \).

Answer: The manager’s payoff is given by
\[
P = \beta P(\alpha + \phi(e) + \epsilon > \bar{\pi}) - \epsilon \\
= \beta \left( 1 - F(\bar{\pi} - \alpha - \phi(e)) \right) - \epsilon
\]
where \( P(x > y) \) is the probability that \( x > y \) and \( F(e) \) is the probability that \( \epsilon \) is less than
\( x \) (cumulative distribution function). Taking the derivative with respect to \( e \), the manager’s
choice of effort level, we get
\[
\frac{dP}{de} = \beta \frac{d\phi(e)}{de} - 1
\]
where \( f(e) \) is the density function of \( \epsilon \). Since \( \mu > \bar{\pi} \), we have \( \mu > \bar{\pi} - \alpha - \phi(e) \). Therefore
\( f(\bar{\pi} - \alpha - \phi(e)) \) is in the increasing portion of \( f \). It follows that an increase in \( \alpha \) leads to
a decrease in \( f(\bar{\pi} - \alpha - \phi(e)) \); and this, in turn, implies a lower \( dP/de \). Finally, a lower
\( dP/de \) implies a lower value of \( \epsilon \). In words, a decrease in the degree of competition (higher
\( \alpha \)) decreases the marginal benefit from managerial effort \( (dP/de) \), and ultimately leads to
a lower effort of managerial effort \( (e) \).
**Applied exercises**

**3.20. Productivity.** Find firm level data on output and inputs (both quantity and prices). Use this information to estimate each firm’s total factor productivity, following the method outlined in Section 3.1 (cf in particular Equation 3.2).